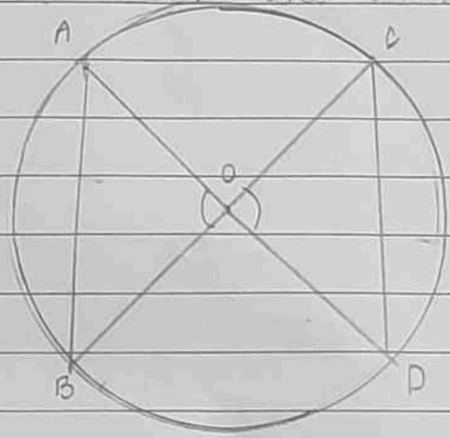
1. A circle is the collection of all points in a plane, which are equidistant from a fixed point in the plane.
2. Equal chords of a circle (or of congruent circles) subtend equal angles at the centre.
3. If the angles subtended by two chords of a circle (or of congruent circles) at the centre (corresponding centres) are equal, the chords are equal.
4. The perpendicular from the centre of a circle to a chord bisects the chord.
5. The line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.
6. There is one and only one circle passing through three non-collinear points.
7. Equal chords of a circle (or of congruent circles) are equidistant from the centre (or corresponding centres).
8. Chords equidistant from the centre (or corresponding centres) of a circle (or of congruent circles) are equal.
9. If two arcs of a circle are congruent, then their corresponding chords are equal and conversely if two chords of a circle are equal, then their corresponding arcs (minor, major) are congruent.
10. Congruent arcs of a circle subtend equal angles at the centre.
11. The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.
12. Angles in the same segment of a circle are equal.
13. Angle in a semicircle is a right angle.
14. If a line segment joining two points subtends equal angles at two other points lying on the same side of the line containing the line segment, the four points lie on a circle.
15. The sum of either pair of opposite angles of a cyclic quadrilateral is 180^0.
16. If sum of a pair of opposite angles of a quadrilateral is 180^0, the quadrilateral is cyclic.

Thm : 1

Statment :

Equal chords of a circle subtend equal angles at the centre.

Diagram :



Given,

In $\odot(O, r)$, AB and CD are equal chords.

To prove : $\underline{\angle AOB} = \underline{\angle COD}$

Proof : In $\triangle AOB$ and $\triangle COD$

AB = CD [Given chords]
OA = OC [radii]
OB = OD [radii]

$\therefore \triangle AOB \cong \triangle COD$ [by SSS congruency]

$\therefore \underline{\angle AOB} = \underline{\angle COD}$ [by CPCT]

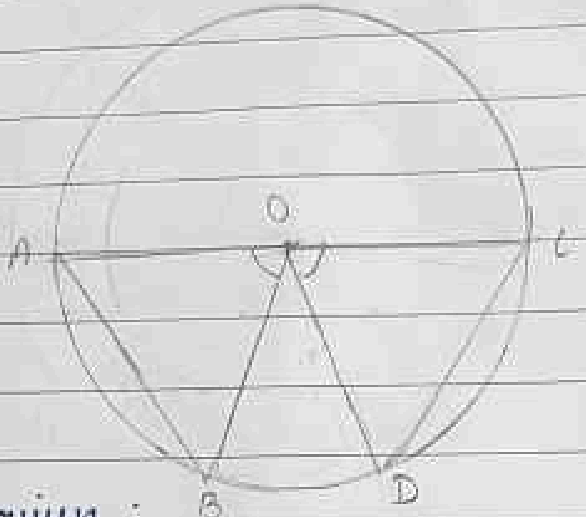
Hence proved.

Thm : ,

Statment :

If the angles subtend by the chords of a circle at the centre are equal, then the chords are equal.

Diagarm :



Given :

In $C(O, r)$,

$\angle AOB = \angle COD$

To prove : $AB = CD$

Proof : In $\triangle AOB$ and $\triangle COD$,

$\angle AOB = \angle COD$ [Given]

$OB = OD$ [radii]

$OA = OC$ [radii]

$\therefore \triangle AOB \cong \triangle COD$ [by SAS congruency]

$\therefore AB = CD$ [by CPCT]

Hence proved.

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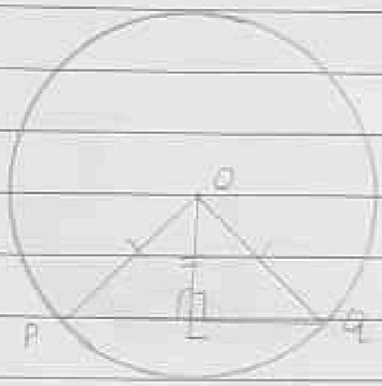
Thm : 3

Statement

The perpendicular from the circle of the circle bisect the cord.

Diagarm



Given,

In $c(O, r)$, PQ is a chord and $OL \perp PQ$

To prove : $PL = LQ$

Proof : In $\triangle OLP$ and $\triangle OLQ$

$OP = OQ$ [radii]

$OL = OL$ [common]

$\angle OLP = \angle OLQ$ [$90°$]

$\therefore \triangle OLP \cong \triangle OLQ$ [by RHS congruency]

$\therefore PL = LQ$ [CPCT]

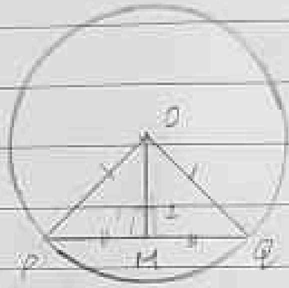
Hence proved.

Thm : 4

Statement

The line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.

Diagram :



Given,

In $C(O, r)$, PQ is a chord and M is its midpoint such that

$PM = MQ$

Construction : Join OP & OQ

To prove : $\angle 1 = \angle 2 = 90°$

Proof : In $\triangle OMP$ and $\triangle OMQ$

$OP = OQ$ [radii]
$PM = MQ$ [$\because$ M is the midpoint]
$OM = OM$ [common]

$\therefore$ $\triangle OMP \cong \triangle OMQ$ [by SSS]

$\therefore \quad \angle 1 = \angle 2 \qquad$ [CPCT]

Now,

$\angle 1 + \angle 2 = 180^\circ \qquad$ [Linear pair]

$$\angle 1 + \angle 1 = 180^\circ$$
$$2\angle 1 = 180^\circ$$
$$\angle 1 = \frac{180}{2}$$
$$\therefore \angle 1 = 90^\circ$$

$$\angle 1 = \angle 2 = 90^\circ$$

Hence proved

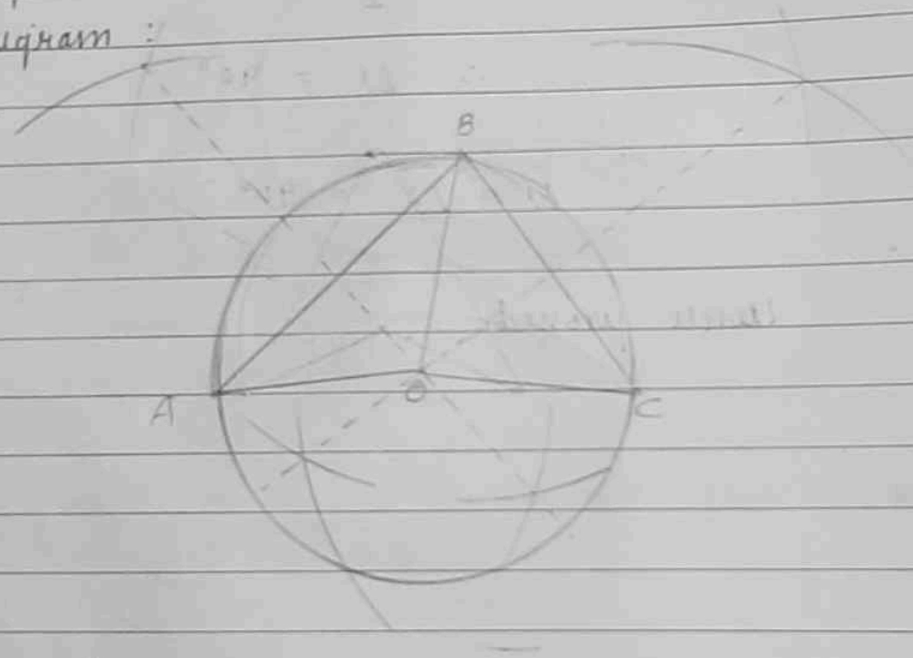
Thm : 5

Statment :

There is one and only one circle passing through three given non-collinear points

~~Diagram~~

Diagram :



Given,

The three non-collinear points are A, B and C

To prove :

There is only one circle passing through A, B and C

Construction :

i) Join AB and CB

ii) Draw perpendicular bisector of AB and BC

iii) These ⊥ bisector meet a point O

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iv) Join OA ~~and~~, OB and OC

v) With O as centre OA as radius draw a circle passing through A, B and C.

Proof :

From the ~~diagonal~~ diagram we get

$OA = OB = OC$ [radii]

Hence we can say that there is only one circle passing through three non-collinear points

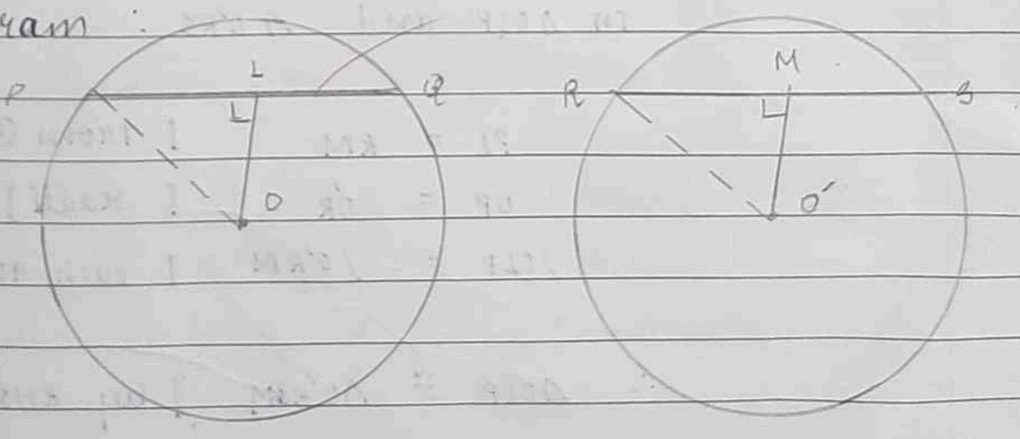
Hence proved

Thm : 6.

Statment :

Equal chords of a circle are equidistand from the centre.

Diagram :



Given,

In $c(O, r)$ and $c(O', r)$, PQ and RS are two equal chords respectively

To prove : $OL = O'M$

Construction : Join OP and $O'R$

Proof :

~~Opposite~~ Since $OL \perp PQ$

$$PL = \frac{1}{2} PQ \quad \text{— ①}$$

Since, $O'M \perp RS$ $RM = \frac{1}{2} RS$ — ②

Also given, $PQ = RS$

$$\div 2 \Rightarrow \frac{1}{2} PQ = \frac{1}{2} RS \quad \text{— ③}$$

Applying ① & ② in ③

$$③ \Rightarrow PL = RM \quad \text{— ④}$$

Now,

In ΔOLP and $\Delta O'RS$

$PL = RM$ [from ④]

$OP = O'R$ [radii]

$\angle OLP = \angle O'RM$ [each 90°]

$\therefore \Delta OLP \cong \Delta O'RM$ [by RHS congruency]

$\therefore OL = O'M$ [by CPCT]

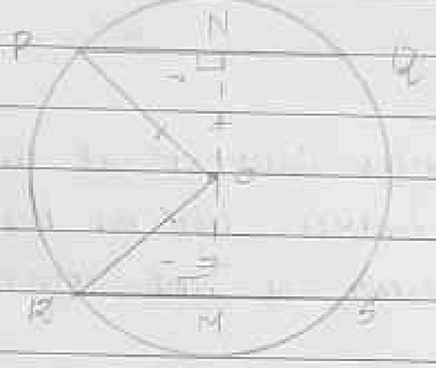
Hence proved .

Thm : 67

Statment :

chords equidistandt from the centre are equal in ~~len~~ length.

Diagram :



Given, In C(O,r),

Two chords PQ and RS are equidistand from the centre such that

OM = ON,

Also ON ⊥ PQ and OM ⊥ RS.

To prove : PQ = RS

Proof : In ΔOPN & ΔORM

OP = OR [radii]

OM = ON [given]

∠1 = ∠2 [each 90°]

∴ ΔORM ≅ ΔOPN [by RHS congruency]

∴ PN = RM [by CPCT]

[∵ OM ⊥ PQ

Now,

$\frac{1}{2}PQ = \frac{1}{2}RS$: OM ⊥ RS]

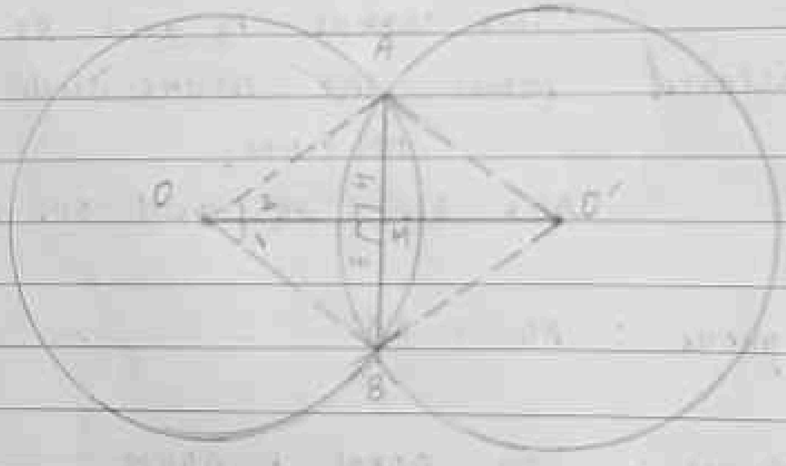
$PQ = RS$

Hence proved.

Exercise 10.3

3. If two circles interest at two points, prove that their centres lie on the perpendicular ~~bisector~~ bisector of the common chord.

Diagram:



Given,

Let C(O, r) and C(O', r') intersect at A and B.

AB is the common chord and OO' is the line segment joining the centre of the O^{les}

Let OO' ~~and~~ and AB intersect eachother at 'M'.

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To prove :

$AM \perp OO'$ and $AM = MB$

Construction :

Join OA, $O'A$, OB and $O'B$.

Proof :

In $\triangle OAO'$ and $\triangle OBO'$

$OA = OB$
$O'A = O'B$ } (radii)
$OO' = OO'$ [common]

$\therefore \triangle OAO' \cong \triangle OBO'$ [by SSS congruency]

$\angle 1 = \angle 2$ [by CPCT]

Now,

In $\triangle OAM$ and $\triangle OBM$,

$OM = OM$ [common]
$OA = OB$ [radius]
$\angle 1 = \angle 2$ [proved]

$\therefore OAM \cong \triangle OBM$ [by SAS]

$\therefore \boxed{AM = MB}$ [by CPCT]

$\angle 3 = \angle 4$ [CPCT]

$$\angle 3 + \angle 4 = 180^\circ \quad \text{[Linear pair]}$$
$$\angle 3 + \angle 3 = 180^\circ$$
$$2\angle 3 = 180^\circ$$
$$\angle 3 = \frac{180}{2}$$
$$\angle 3 = 90^\circ$$
$$\therefore \angle 3 = \angle 4 = 90^\circ$$
$$\therefore AM \perp OO'$$

Hence proved.

<u>Notes</u>

1. Equal chords of a circle subtend equal angles at the centre and viceversa

2. The line drawn through the centre of a circle to bisect a chord is perpendicular to the cord and vice-versa

3. Equal chords of a circle are equidistant from the ~~centre of the~~ circle

4. Chords corresponding to equal arcs are equal.

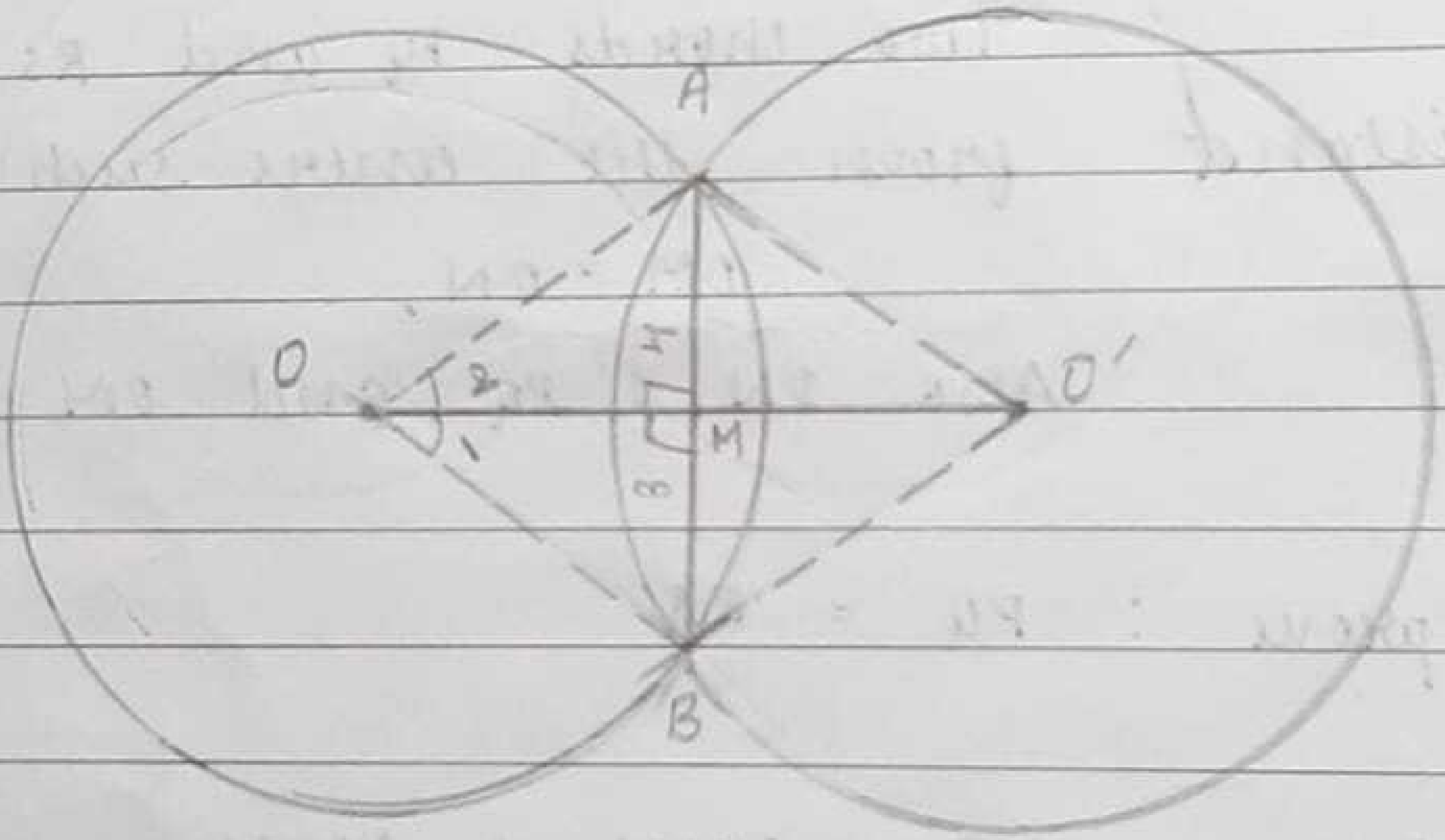
5. Congruent arcs of a circle subtend equal angles at the centre

6. The angle subtended by ~~an~~ an arc at the centre is <u>double</u> the angle subtended by it at any point on the remaining part of the circle.

Exercise 10.3

3. If two circles interest at two points, prove that their centres lies on the perpendicular ~~bisector~~ bisector of the common chord.

Diagram :



Given,

Let $c(O, r)$ and $c(O', r')$ intersect at A and B.

AB is the common chord and OO' is the line segment joining the centre of the O^{les}

Let OO' ~~and~~ and AB intersect eachother at 'M'.

To prove :

$AM \perp OO'$ and $AM = MB$

construction :

Join OA, $O'A$, OB and $O'B$.

Proof :

In $\Delta OAO'$ and $\Delta OBO'$

$$\left.\begin{array}{l} OA = OB \\ O'A = O'B \end{array}\right\} \text{(radii)}$$

$OO' = OO'$ [common]

$\therefore \Delta OAO' \cong \Delta OBO'$ [by SSS congruency]

$\angle 1 = \angle 2$ [by CPCT]

Now,

In ΔOAM and ΔOBM,

$OM = OM$ [common]

$OA = OB$ [radius]

$\angle 1 = \angle 2$ [proved]

$\therefore OAM \cong \Delta OBM$ [by SAS]

$\therefore \boxed{AM = MB}$ [by CPCT]

$\angle 3 = \angle 4$ [CPCT]

$\angle 3 + \angle 4 = 180^\circ$ [Linear pair]

$\angle 3 + \angle 3 = 180^\circ$

$2\angle 3 = 180^\circ$

$\angle 3 = \frac{180}{2}$

$\angle 3 = 90^\circ$

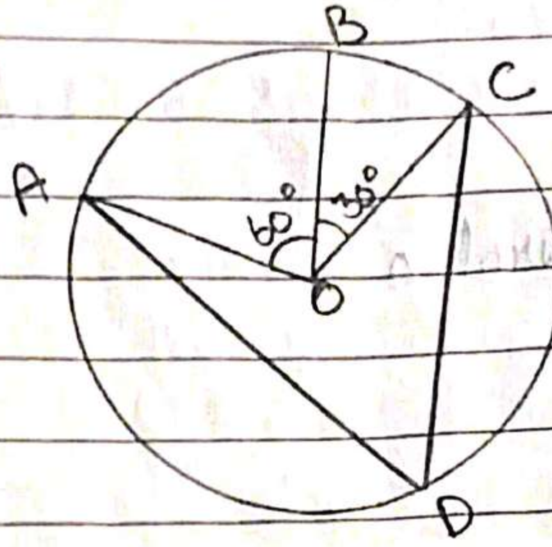
$\therefore \angle 3 = \angle 4 = 90^\circ$

$\therefore AM \perp OO'$

Hence proved.

Ex: 10.5

1. Refer text book pg: 184



given:

$\angle BOC = 30°$

$\angle AOB = 60°$

$\angle AOC = 60° + 30°$

$= 90°$

$\angle AOC = 2(\angle ADC)$ (Theorem : 8)

$\Rightarrow \angle ADC = \frac{1}{2} \angle AOC$

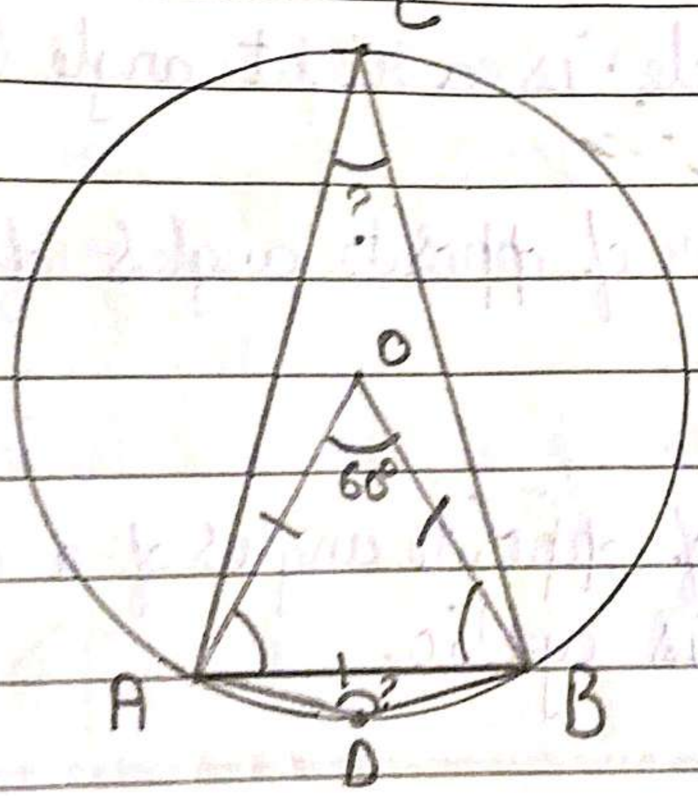
$\angle ADC = \frac{1}{\cancel{2}_1} \times \cancel{90}^{45}°$

$\angle ADC = 45°$

$\angle ADC = 45°$

$\boxed{\text{Ans} \Rightarrow \angle ADC = 45°}$

2. Refer Text book pg: 185



Soln:

given :

$OA = OB = AB$

$\therefore$ OAB is an equilateral Δ

$\Rightarrow \angle AOB = 60^\circ$.

i) Minor arc :

$\angle AOB = 2\angle ACB$

$\angle ACB = \frac{1}{2} AOB$

$= \frac{1}{\cancel{2}} \times \cancel{60}^{30}$

$\boxed{\therefore \angle ACB = 30^\circ}$

ii) Major arc :

Reflex $\angle AOB = 360^\circ - 60^\circ$

$= 300^\circ$

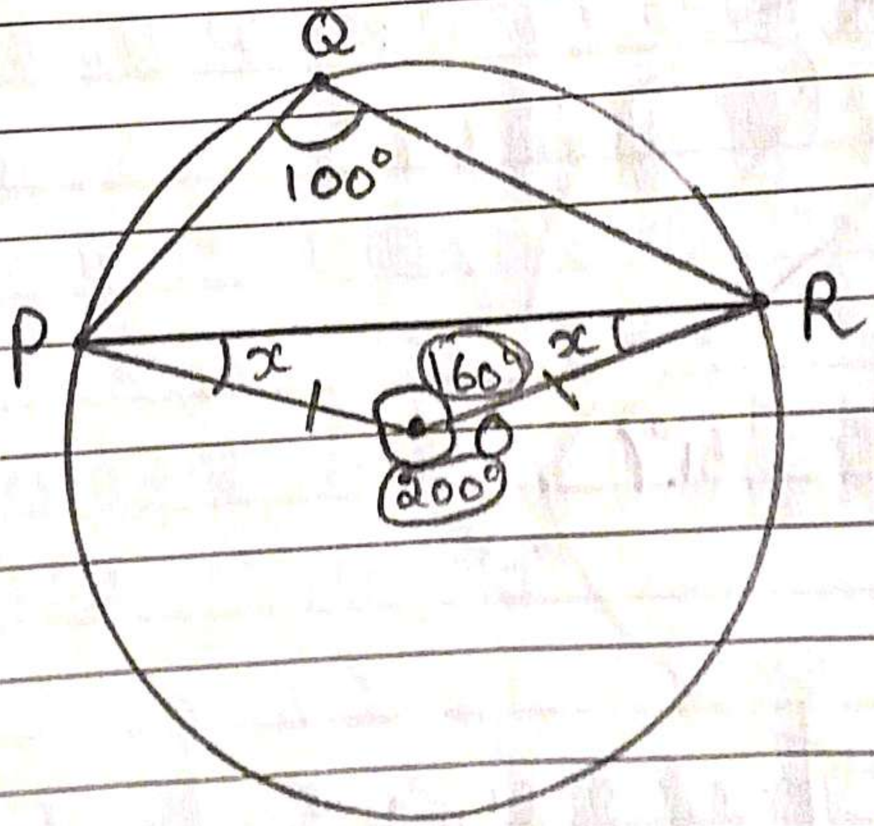
$\therefore \angle ADB = \frac{1}{2}$ reflex $\angle AOB$

$= \frac{1}{\cancel{2}} \times \cancel{300}^{150^\circ}$

$\boxed{\therefore \angle ADB = 150^\circ}$

$\boxed{\text{Ans} \Rightarrow \angle ACB = 30^\circ, \angle ADB = 150^\circ}$ ✓

5. Refer Text book pg: 185



Soln:

Soln:

given:

$\angle PQR = 100^\circ$

$\because$ PR is a major arc (Theorem 8 and

Reflex $\angle POR = 2 \angle PQR$

$\therefore$ Reflex $\angle POR = 2 \times 100$

Reflex $\angle POR = 200^\circ$

$\therefore$ $\angle POR =$ ~~Ref~~ $360^\circ -$ Reflex $\angle POR$

$= 360^\circ - 200^\circ$

$\angle POR = 160^\circ$

$\angle POR = 160^\circ$

In $\triangle POR$,

$x + x + \angle POR = 180^\circ$ (Angle Sum Property

$\Rightarrow 2x + 160^\circ = 180^\circ$

$x = \dfrac{180^\circ - 160^\circ}{2}$

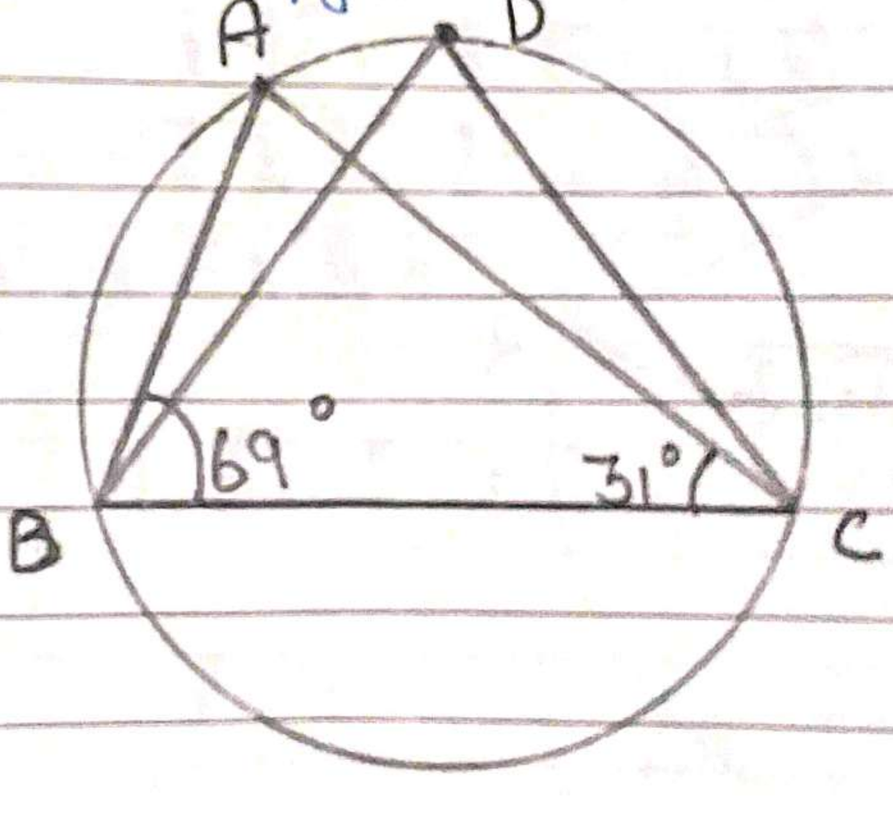
$x = \dfrac{\cancel{20}^{10}}{\cancel{2}}$

$\therefore x = 10^\circ$

$\Rightarrow \angle OPR = \angle ORP = 10^\circ$

$\boxed{\text{Ans} \Rightarrow \angle OPR = 10^\circ}$ ✓

4. Refer Text Book pg: 185



Soln:

given:

In $\triangle ABC$,

$\angle A + \angle B + \angle ACB = 180^\circ$ (∵ Angle Sum Property of a $\triangle$)

$\Rightarrow \angle A + 69^\circ + 31^\circ = 180^\circ$

$\Rightarrow \angle A = 180^\circ - 100^\circ$

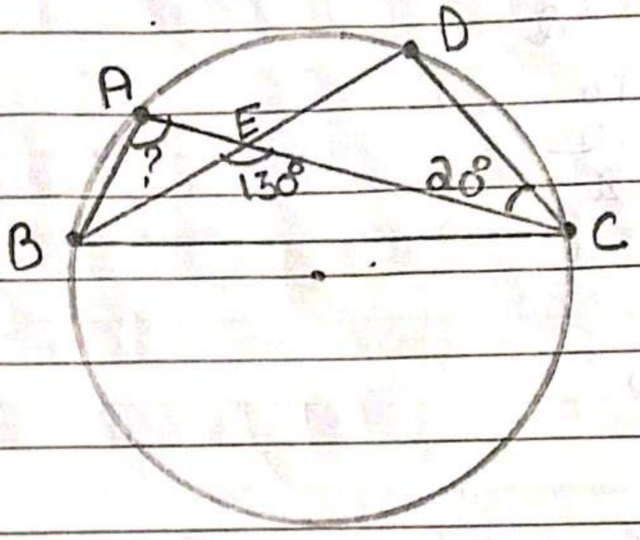
$\angle A = 80^\circ$

∵ $\angle A = \angle D = 80^\circ$ (Angles on the Same Segment ar[e] equal.)

∴ $\angle BDC = 80^\circ$

$\boxed{\text{Ans} \Rightarrow \angle BDC = 80^\circ}$ ✓

5. Refer Text book pg : 185



Soln :

given:

$\angle BEC = 130°$

$\angle DCE = 20°$

$\angle BEC + \angle DEC = 180°$ (Linear Pair)

$\Rightarrow 130° + \angle DEC = 180°$

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$\Rightarrow \quad \angle DEC = 180^\circ - 130^\circ$

$\therefore \quad \angle DEC = 50^\circ$

In ΔDEC,

$\angle DEC + \angle DCE + \angle EDC = 180^\circ$ (Angle Sum Property of a Δ)

$\Rightarrow \quad 50^\circ + 20^\circ + \angle EDC = 180^\circ$

$\Rightarrow \quad 70^\circ + \angle EDC = 180^\circ$

$\Rightarrow \quad \angle EDC = 180^\circ - 70^\circ$

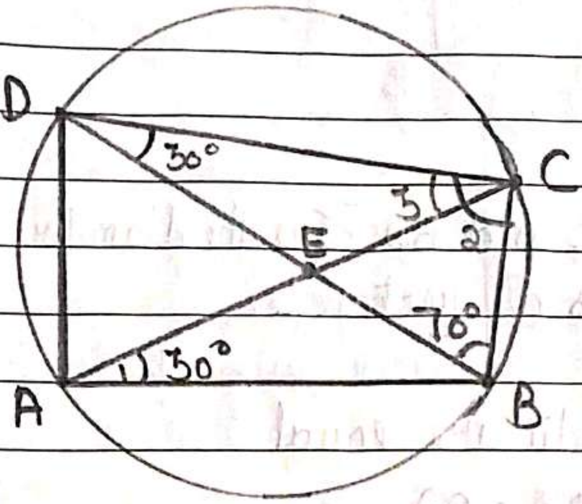
$\therefore \quad \angle EDC = 110^\circ$

$\therefore \quad \angle BAC = \angle EDC = 110^\circ$ ($\because$ Angles on the same segment are equal

$\therefore \quad \angle BAC = 110^\circ$

$\boxed{\text{Ans} \Rightarrow \angle BAC = 110^\circ}$ ✓

6. Refer Text Book pg: 186



Soln:

given:

ABCD is a cyclic quadrilateral

$\angle DBC = 70°$

$\angle BAC = 30°$

Diagonals AC and BD intersect at point E.

$\angle BAC = \angle BDE = 30°$ (Angles on the line segment are equal.)

In ΔBCD

$\angle BCD + \angle BDC + \angle CBD = 180°$ (Angle Sum Property of Δ)

$\Rightarrow \angle BCD + 30° + 70° = 180°$

$\angle BCD = 180° - 100°$

$\Rightarrow \angle BCD = 80°$

given: $AB = BC$,

$\therefore \angle ECB = \angle BAE = 30^\circ$ (Angles opposite to equal sides are equal.)

$\angle BCD = \angle BCE + \angle ECD$

$80^\circ = 30^\circ + \angle ECD$

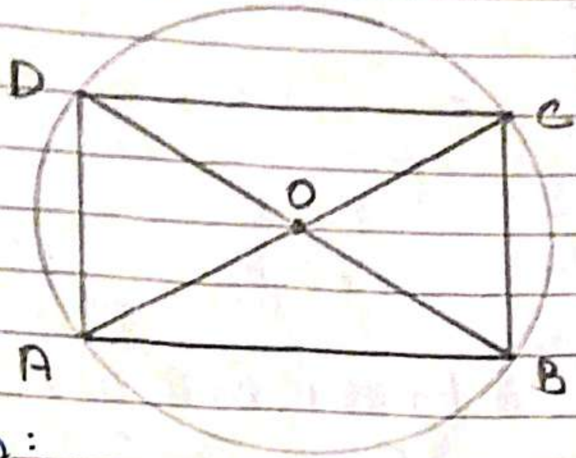
$\Rightarrow \angle ECD = 80^\circ - 30^\circ$

$= 50^\circ$

$\therefore \angle ECD = 50^\circ$

$\boxed{\text{Ans} \Rightarrow ECD = 50^\circ}$

Refer text book pg: 125



given:

Diagonals AC and BD are the diameters of a circle

To prove: ABCD is a rectangle

Proof:

$\because$ all the radii are equal

$OA = OC = OB = OD$

$\Rightarrow \quad OA = OC = \frac{1}{2} AC$ —— (1)

$\Rightarrow \quad OB = OD = \frac{1}{2} BD$ —— (2)

From (1) and (2)

$\cancel{\frac{1}{2}} AC = \cancel{\frac{1}{2}} BD$

$\Rightarrow \quad AC = BD$

Thus, the diagonals of a quadrilateral are equal

Hence ABCD is a rectangle.

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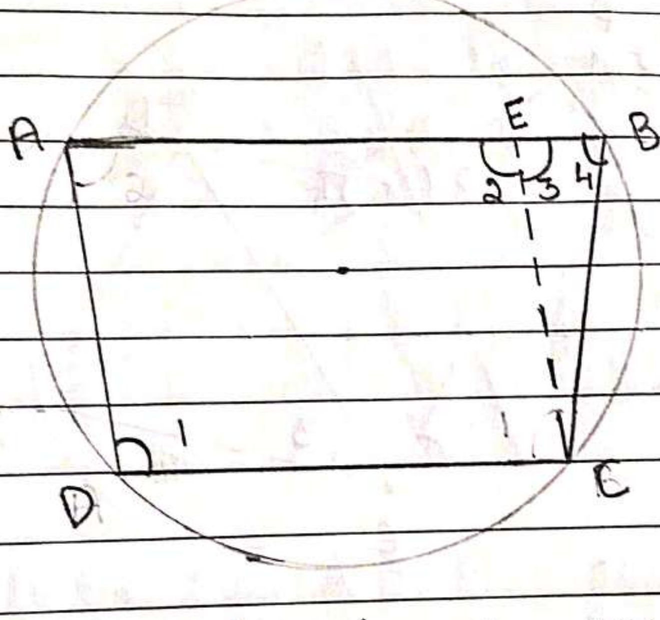
8. Prove that an isoscles trapezium is always cyclic or If the Non-parallel sides of a trapezium are equal, Prove that it is cyclic.



given :

ABCD is a traperium in which,

AB ∥ CD , and

AD = BC

AB ∥ CD , and

AD = BC

To prove :

Traperium ABCD is a cyclic

Construction:

Draw CE ∥ AD

Proof:

∵ AB∥CD , and (given)

AD∥CE (construction)

∴ ADCE is a ∥gm

⇒ ∠1 = ∠2 (Opposite angles of a ∥gm are equal.) —— ①

But AD = CB (given)

⇒ CE = CB (∵ AD = CE)

∴ ∠3 = ∠4 (Angles opposite to equal sides are equal.) —— ②

Also,

∠2 + ∠3 = 180° (Linear Pair)

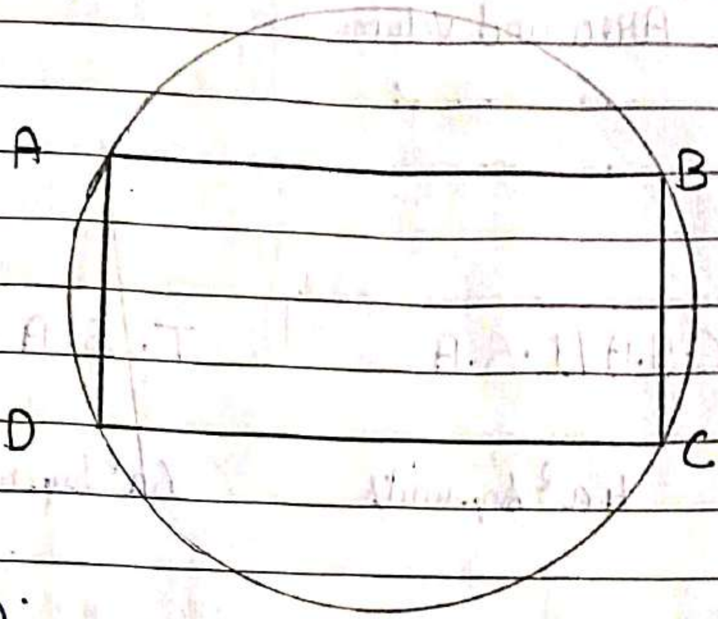
⇒ ∠1 + ∠4 = 180° (Using ① and ②)

(i.e) ∴ Sum of Opposite angles of a trapezium ABCD is 180°

Hence, ABCD is a cyclic

Hence Proved.

12. Refer text book pg: 186



given:

∥gm ABCD inscribed in a circle

To prove:

ABCD is a rectangle

Proof:

$\because$ ABCD is a cyclic quadrilateral (∥gm)

$\therefore \angle A + \angle C = 180°$ ——— ①

Also, ABCD is a ∥gm

$\therefore \angle A = \angle C$ ——— ②

From ① and ②

$\angle A = \angle C = 90°$

Similarily,

$\angle B = \angle D = 90°$

Thus in a ∥gm ABCD,

$\angle A = \angle B = \angle C = \angle D = 90°$

Hence ABCD is a rectangle.

Hence proved