Chapter-5
Introduction To Euclids Geometry

Exercise 5.1

1) Which of the following statements are true and which are false? Give reasons for your answers.

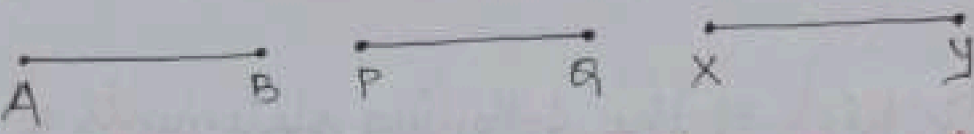
i) Only one line can pass through a single point. -False [∵ Infinite lines can pass through a single point].

ii) There are an infinite number of lines which pass through two distinct points. -False [Axiom -"Given two distinct points, there is a unique line that passes through them".].

iii) A terminated line can be produced indefinitely on both the sides. -True [Postulate -"A terminated line can be produced indefinitely"].

iv) If two circles are equal, then their radii are equal. -True [∵ Only when the centres and radii of the circles superimpose on each other, we can say that the circles are equal.

v) In the below figure, if $AB = PQ$ and $PQ = XY$, then $AB = XY$.



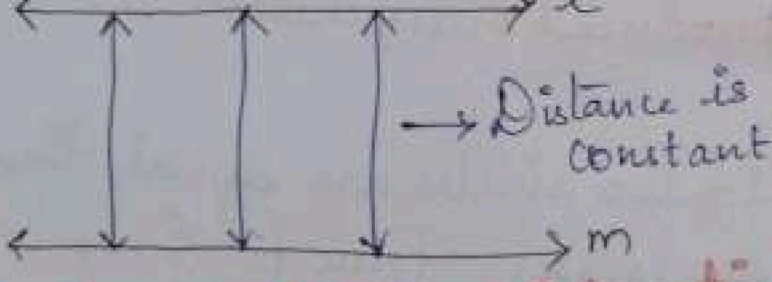
True [$\because$ Axiom - Things which are equal to one same thing, are equal to one another.]

2) Give a definition for each of the following terms. Are there other terms that need to be defined first? What are they and how might you define them?

i) Parallel lines.

⟹ Parallel lines are those lines which do not intersect at any point. Also, the distance between the parallel lines will be always constant.

⇒

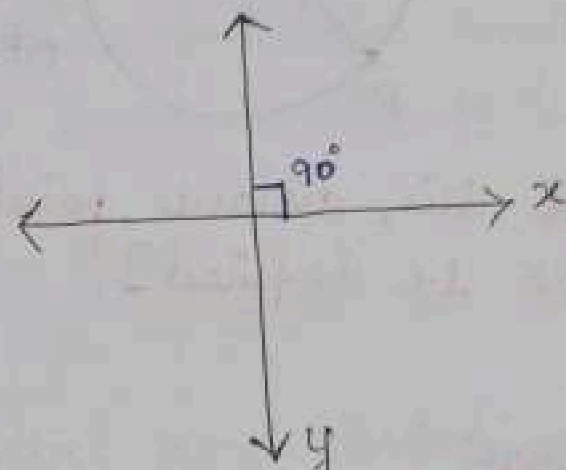


[For this, lines and point of intersection should be defined].

ii) Perpendicular lines

⇨ When the lines intersect at 90°, then they are called perpendicular lines.

⇒

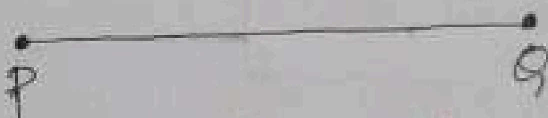


[For this lines, point of intersection and angle are to be defined].

iii) Line segment.

⇨ A straight line draw from one point to another point is called as a line segment

⇒

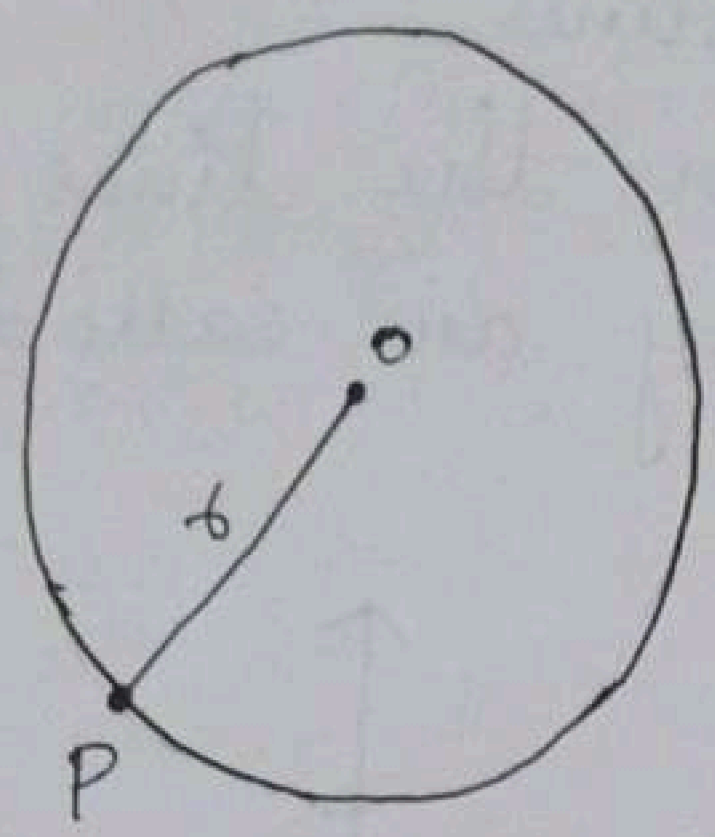


[For this points should be defined].

iv) Radius of a circle.

⇨ The distance between the circle and any point lying on the circle is called the radius of the circle.

⇒



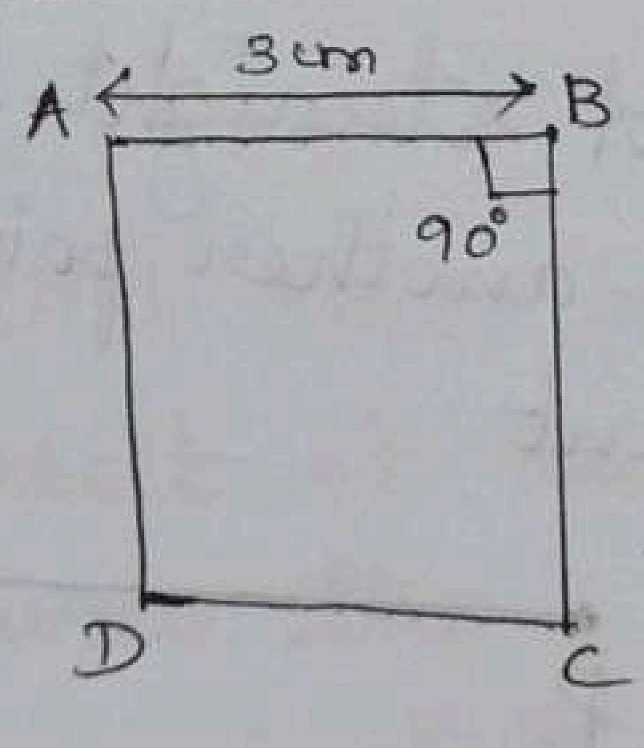
$O \to$ centre
$P \to$ point on the circle
$OP = r \to$ radius.

[For this, distance, point, centre, circle are to be defined].

v) Square

⇒ The quadrilateral in which all the four sides are equal and all the four angles are 90° then it is called as a square.

⇒



[For this, quadrilateral, angle, sides are to be defined].

3) Consider two postulates given below:

i) Given any two distinct points A and B, there exists a third point C which is in between A and B.

ii) There exist at least three points that are not on the same line.

Do these postulates contain any undefined terms? Are these postulates consistent? Do they follow from Euclid's postulates? Explain.

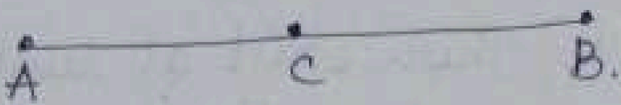
Solution :-

* There are undefined terms in the given postulates.
* The given postulates are consistent because, they refer to two different situations. Also, there is no axiom and postulate to contradict the given statements.
* They donot follow any postulate but an axiom can be related to them.

"Given two distinct points, there is a unique line that passes through them".

4) If a point C lies between two points A and B such that $AC = BC$, then prove that $AC = \frac{1}{2}AB$. Explain by drawing the figure.

Solution:-



Given- $AC = BC$, $AC + BC = AB$.

Adding AC on both sides,

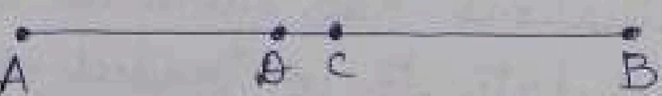
(Axiom 2) $\rightarrow AC + AC = BC + AC$

$$2AC = AB$$

$$AC = \frac{1}{2} AB.$$

5) In question 4, point C is called a midpoint of linesegment AB. Prove that every line segment has one and only one mid-point.

Solution:-



Given- C is the midpoint of AB.

If possible, let D be the midpoint of AB.

$AC = CB$ [$\because$ C is the midpoint]

Adding AC on both sides,

$AC + AC = CB + AC$. [Axiom 2]

$\Rightarrow AC + AC = AC + CB$

$2AC = AB \longrightarrow (1)$

$AC = \frac{1}{2} AB.$

If D is the midpoint,

$$AD = DB$$

Adding AD on both sides,

$$AD + AD = AD + DB$$

$$2AD = AB \longrightarrow (2)$$

$$AD = \frac{1}{2} AB$$

From, (1) and (2),

$$2AC = 2AD$$

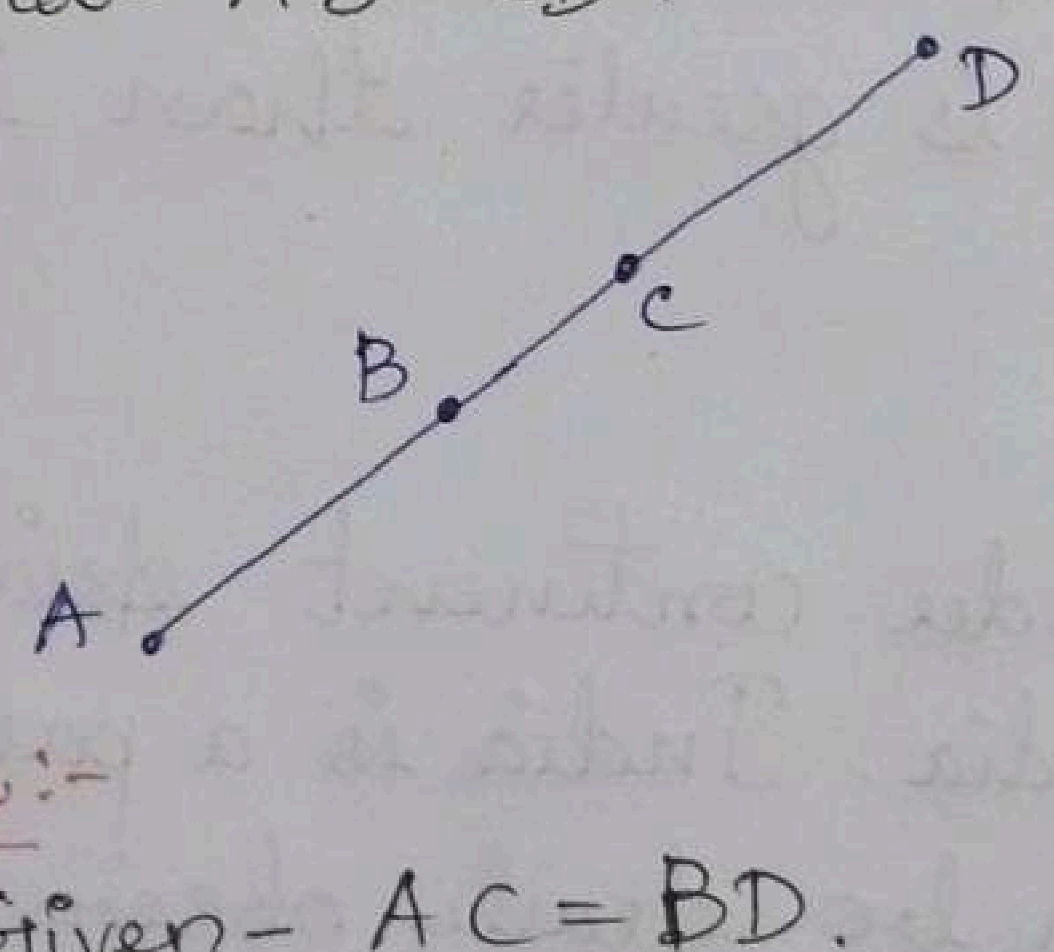
$$AC = AD.$$

This means that both points C and D are same.

Hence our assumption is wrong

$\therefore$ Every line segment has one and only one mid-point.

6) In the below figure, if $AC = BD$, then prove that $AB = CD$.



Solution:-

Given- $AC = BD$.

From the figure,

$$AC = AB + BC$$

and $BD = BC + CD$.

$\therefore AB + \cancel{BC} = \cancel{BC} + CD$ [$\because AC = BD$ Given]

$\therefore \boxed{AB = CD}$.

Hence Proved.

7) Why is Axiom 5, in the list of Euclid's axioms, considered a 'universal truth'?

Solution :-

→ Axiom 5 states that,

" The whole is greater than the part"

→ The axiom is known as Universal truth because it is true in any field and not just in Mathematics.

→ Let us take 2 cases as examples.

Case 1 :-

$14 = 2 + 7 + 5$

Clearly, $14 > 2$, $14 > 7$, $14 > 5$ i.e., whole (14) is greater than its parts (2, 7, 5).

Case 2 :-

Consider continent Asia and country India. India is a part of Asia and it can be clearly observed that Asia is greater than India which again makes Axiom 5 to be true.

$\therefore$ Axiom 5 is a 'Universal Truth'.