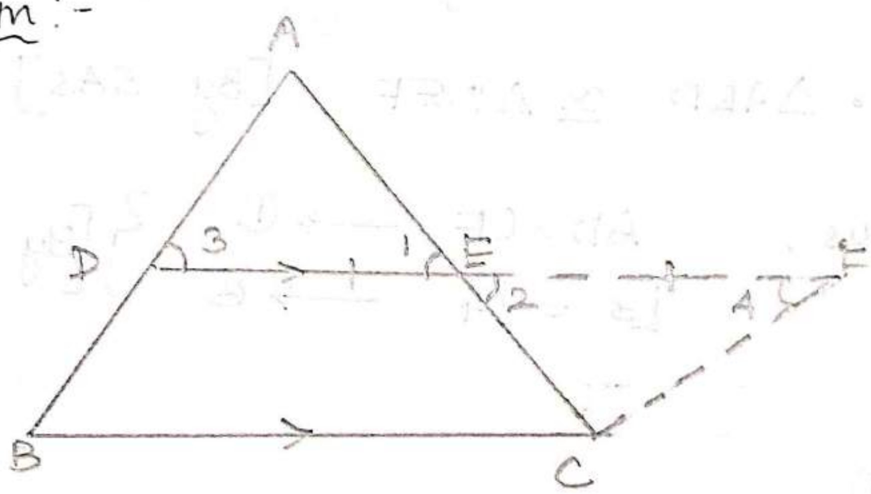
MID-POINT THEOREM

State and Prove Mid-point theorem.

Statement :-

The line segment joining the midpoint of any two sides of a triangle is parallel to the third side and half of it.

Diagram :-



Given :-

In ΔABC,

D is the midpoint of AB &

E is the midpoint of AC

DE is joined.

To prove :-

(i) $DE \parallel BC$

(ii) $DE = \frac{1}{2} BC$

Construction :-

Produce DE to F, such that

DE = EF. Join FC.

Proof :-

In $\triangle AED$ and $\triangle CEF$

$AE = EC$ [Given]

$\angle 1 = \angle 2$ [Vertically opposite angles]

$DE = EF$ [By construction]

$\therefore \triangle AED \cong \triangle CEF$ [By SAS]

Thus, $AD = CF \longrightarrow (1)$
$\angle 3 = \angle 4 \longrightarrow (2)$ } [By CPCT]

From (1),

$\Rightarrow AD = CF$

Since D is the midpoint, $AD = DB$

$\Rightarrow \boxed{BD = CF} \longrightarrow (3)$

From (2),

$\Rightarrow \angle 3 = \angle 4$ [Alternate int. angles]

$\Rightarrow AD \parallel CF$

$\Rightarrow \boxed{BD \parallel CF} \longrightarrow (4)$

From (3) & (4),

One pair of opposite sides are equal and parallel,

$\therefore$ BCFD is a parallelogram.

Hence, $DF \parallel BC$ and $DF = BC$.

Since $DF \parallel BC$, DE is also $\parallel$ BC.

$\Rightarrow DE \parallel BC$ [proved (i)]

Now, $DF = BC$

$\div 2 \Rightarrow \frac{DF}{2} = \frac{BC}{2}$

$DE = \frac{BC}{2}$ [proved (ii)]

$\therefore DE \parallel BC$ & $DE = \frac{BC}{2}$

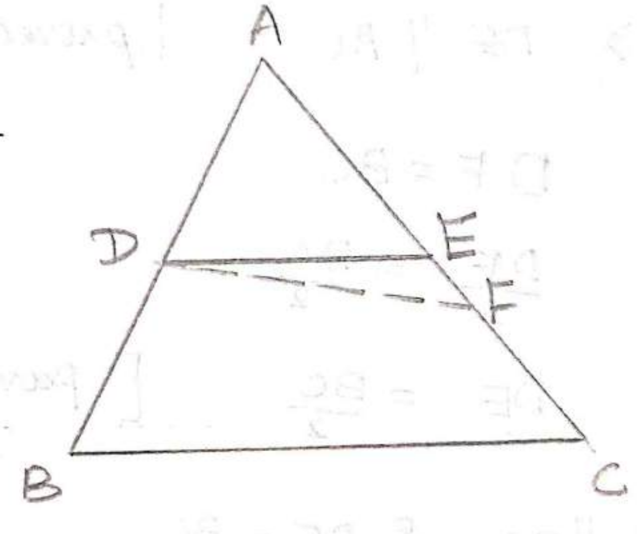
Hence proved.

Converse of Mid-point theorem:

Statement:-

The line drawn through the mid-point of one side of a triangle, parallel to another side, bisects the third side

Diagram:-



Given:-

In ΔABC,

D is the midpoint of AD. & $DE \parallel BC$.

To prove:-

E is the mid-point of AC.

Proof:-

Let us assume E is not the mid-point of AC.

Let "F" be the mid-point of AC.

Join DF.

In ΔABC,

D is the mid-point of AB.

and F is the mid-point of AC.

$\therefore$ By Mid-point thm,

$DF \parallel BC \longrightarrow (1)$.

Also given that $DE \parallel BC \longrightarrow (2)$

From (1) & (2),

Two intersecting lines DE and DF are parallel to BC.

This is possible only if E and F coincide each other.

$\therefore$ Our assumption is wrong.

Thus, E is the midpoint of AC.

Hence proved.