Chapter - 1
Number System.

Notes:

* Natural numbers (N) : $\{1, 2, 3, \ldots\}$
* Whole numbers (W) = $\{0, 1, 2, 3, \ldots\}$
* Integers (Z) = $\{\ldots -2, -1, 0, 1, 2, \ldots\}$
* Rational Numbers (Q) = $\{\frac{p}{q} ; p, q \in Z \;\&\; q \neq 0\}$

belongs to

Irrational Numbers:

* An Irrational number cannot be written in the form $\frac{p}{q}$ where p & q are integers and $q \neq 0$.

Eg: $\sqrt{2}, \sqrt{3}, \pi$

* Irrational numbers are represented by non-terminating and non-repeating decimals.
* There are infinitely many rational numbers between any 2 given rational numbers.

Real numbers (R):

* Rational and Irrational Numbers taken together to form the set of real numbers.

Exercise:

1) Find a rational number between 3 & 4.

Rational number between 3 & 4 $= \frac{3+4}{2}$

$= \frac{7}{2}$

$= 3.5$

$\boxed{\text{Ans} = 3.5}$

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2. Find a rational number between $\frac{-1}{3}$ & $\frac{1}{4}$

Soln:

A rational number between $\frac{-1}{3}$ & $\frac{1}{4}$ $= \dfrac{\frac{-1}{3} + \frac{1}{4}}{2}$

$$= \left(\frac{-4+3}{12}\right) \div 2$$

$$= \left(\frac{-1}{12}\right) \div 2$$

$$= \frac{-1}{12} \times \frac{1}{2} = \frac{-1}{24}$$

$\therefore$ A rational number between $\frac{-1}{3}$ & $\frac{1}{4}$ $= \frac{-1}{24}$

3. Find 3 rational numbers between -2 & 5

$\therefore$ 3 Rational numbers between -2 & 5 are $1, 2, 3$

4. Find 5 rational number between $\frac{3}{4}$ & $\frac{7}{4}$

$$\frac{3}{4} = \frac{3 \times 6}{4 \times 6} = \frac{18}{24}$$

$$\frac{7}{4} = \frac{7 \times 6}{4 \times 6} = \frac{42}{24}$$

$\therefore$ 5 Rational numbers between $\frac{3}{4}$ & $\frac{7}{4}$ are $\frac{19}{24}, \frac{20}{24}, \frac{21}{24}, \frac{22}{24}, \frac{23}{24}$

H.W

5. Find 6 rational numbers between $\frac{3}{5}$ & $\frac{4}{5}$

$$\frac{3}{5} = \frac{3 \times 7}{5 \times 7} = \frac{21}{35}$$

$$\frac{4}{5} = \frac{4 \times 7}{5 \times 7} = \frac{28}{35}$$

$\therefore$ 6 Rational numbers between $\frac{3}{5}$ & $\frac{4}{5}$ are

$\frac{22}{35}, \frac{23}{35}, \frac{24}{35}, \frac{25}{35}, \frac{26}{35}, \frac{27}{35}$.

II. Write the following
(i) in decimal
(ii) The kind of decimal expansion.
(iii) length
(iv) Period.

1) $\frac{5}{8}$

$$
\begin{array}{r|l}
 & 0.625 \\
8 & 5.000 \\
 & 0 \\ \hline
 & 50 \\
 & 48 \\ \hline
 & 20 \\
 & 16 \\ \hline
 & 40 \\
 & 40 \\ \hline
 & 0
\end{array}
$$

(i) $\frac{5}{8} = 0.625$

(ii) Terminating decimal.

2) $\frac{20}{3}$

$$
\begin{array}{r|l}
 & 6.666 \\
3 & 20.000 \\
 & 18 \\ \hline
 & 20 \\
 & 18 \\ \hline
 & 20 \\
 & 18 \\ \hline
 & 20 \\
 & 18 \\ \hline
 & 2
\end{array}
$$

(i) $\frac{20}{3} = 6.66....$ (or) $6.\overline{6}$

(ii) It is non-terminating but repeating decimal.

(iii) 1

(iv) 6

3) $\frac{100}{11}$

$$
\begin{array}{r}
9.0909 \\
11 \overline{)\,100.0000} \\
\underline{99} \\
10 \\
\underline{0} \\
100 \\
\underline{99} \\
10 \\
\underline{0} \\
100 \\
\underline{99} \\
1
\end{array}
$$

(i) $\frac{100}{11} = 9.\overline{09}$

(ii) Non Terminating ; repeating decimal.

(iii) 2

(iv) 09

III. Conversition of decimals into $\frac{p}{q}$ form. (or) $\frac{m}{n}$

1) Express $0.\overline{6}$ in the form $\frac{p}{q}$

Soln:

$0.\overline{6} = 0.666.....$

Let $x = 0.666.....$ ——— (1)

(1) $\times 10$

$\Rightarrow 10x = 6.666.....$ ——— (2)

(2) $-$ (1)

$\Rightarrow 10x - 1x = (6.666....) - (0.666....)$

$\Rightarrow 9x = 6$

$x = \frac{6}{9} = \frac{2}{3}$

$$\boxed{0.\overline{6} = \frac{2}{3}}$$

2) Represent $0.\overline{23}$ in the form $\frac{m}{n}$

Soln:

$$0.\overline{23} = 0.2323\ldots$$

Let $x = 0.2323\ldots$ —— ①

① × 100

$$\Rightarrow 100x = 23.2323\ldots \quad —— ②$$

$23.2323\ldots$
0.2

② − ①

$$\Rightarrow 100x - x = (23.2323\ldots) - (0.2323\ldots)$$

$$\Rightarrow 99x = 23$$

$$x = \frac{23}{99}$$

$$\boxed{0.\overline{23} = \frac{23}{99}}$$

3) Express $18.\overline{48}$ in $\frac{p}{q}$ form.

Method: I

$$18.\overline{48} = 18.4848\ldots$$

$$18.\overline{48} = 18 + 0.4848\ldots$$

$$= 18 + \frac{48}{99}$$

$$= \frac{1782+48}{99}$$

$$18.\overline{48} = \frac{\cancel{1830}^{610}}{\cancel{99}_{33}}$$

$$\boxed{18.\overline{48} = \frac{610}{33}}$$

4. Express $0.2\overline{35}$ in the form $\frac{m}{n}$

Method: I

Let $x = 0.2\overline{35}$ —— ①

① × 10 $\Rightarrow 10x = 2.3535\cdots$ —— ②

$\Rightarrow 10x = 2 + 0.3535\cdots\cdots$

$\Rightarrow 10x = \frac{2}{1}^{\times 99} + \frac{35}{99}$

$\Rightarrow 10x = \frac{198+35}{99}$

$\Rightarrow 10x = \frac{233}{99}$

$x = \frac{233}{990}$

$$\boxed{0.2\overline{35} = \frac{233}{990}}$$

METHOD: 2

Let $x = 0.23535\ldots$ —— ①

① X10

$\Rightarrow 10x = 2.3535\ldots$ —— ②

② X100

$\Rightarrow 1000x = 235.3535\ldots$ —— ③

③ - ②

$\Rightarrow 1000x - 10x = (235.3535\ldots) - (2.3535\ldots)$

$\Rightarrow 990x = 233$

$\Rightarrow x = \frac{233}{990}$

$$\boxed{0.2\overline{35} = \frac{233}{990}}$$

HW

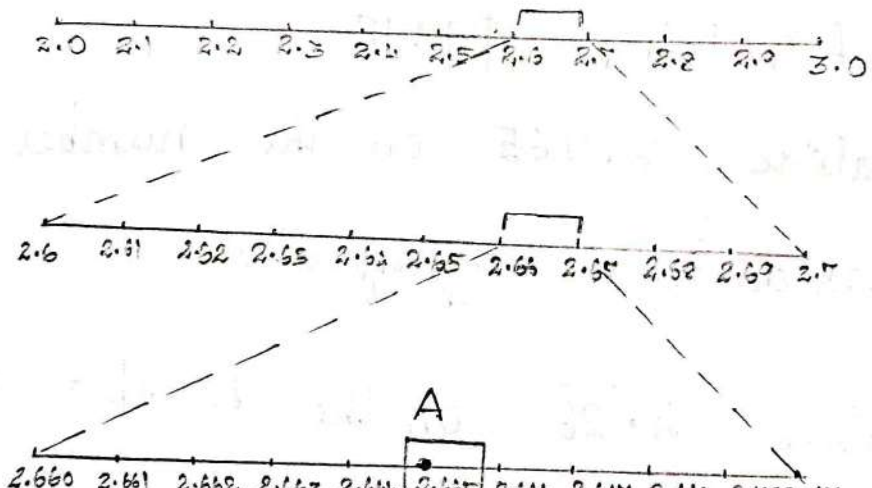
5) Express $0.4\overline{7}$ in the form $\frac{p}{q}$.

6) Convert $0.\overline{001}$ into the form p/q.

Representing Real Numbers on the Number Line.

1. Visualise 2.665 on the number line, using successive magnification.

soln:-



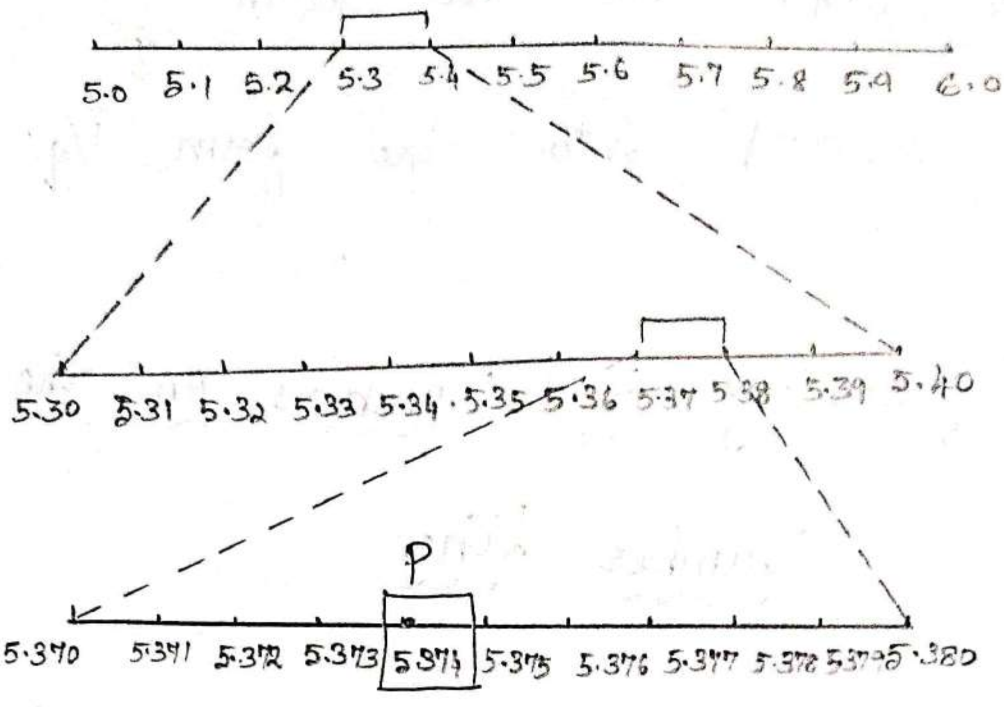
Point 'A' represents the given rational number.

2. Visualise $5.\overline{37}$ on the number line.

soln:

$5.\overline{37} = 5.3737$

$\simeq 5.374$ [Rounded off]



∴ Point P represents the given rational number.

HW.

EX 1.4 Pg : 18.

1) Visualise 3.765 on the number line, using successive magnification.

2) Visualise $4.\overline{26}$ on the number line, upto 4 decimal places.

1. Show that $\sqrt{2}$ is an irrational number by contradiction method

Proof:

Let us assume $\sqrt{2}$ is a rational number.

$\therefore \sqrt{2} = \frac{p}{q}$ [where p and q are co-prime integers and $q \neq 0$.]

squaring on both sides,

$$(\sqrt{2})^2 = \left(\frac{p}{q}\right)^2$$

$$2 = \frac{p^2}{q^2}$$

$$2q^2 = p^2$$

$$\Rightarrow \boxed{p^2 = 2q^2} \longrightarrow (1)$$

ie, p^2 is divisible by 2.

$\therefore$ p is also divisible by 2. $\longrightarrow$ (2)

Let $p = 2m$

Squaring on both sides,

$$p^2 = (2m)^2$$

$$2q^2 = 4m^2 \qquad [\because p^2 = 2q^2 \text{ by } (1)]$$

$$q^2 = \frac{\cancel{4}^{2}m^2}{\cancel{2}}$$

$$\boxed{q^2 = 2m^2} \longrightarrow (3)$$

ie, q^2 is divisible by 2.

$\therefore$ q is also divisible by 2. $\longrightarrow$ (4)

From ② & ④,

p and q are divisible by 2.

$\therefore$ p and q have a common factor 2, which is contradicts our assumption.

$\therefore$ Our assumption is wrong.

Thus, $\sqrt{2}$ is an irrational number.

Hence proved.

2. Prove that $\sqrt{5}$ is an irrational number by contradiction method.

Proof:

Let us assume $\sqrt{5}$ is a rational number.

$\therefore \sqrt{5} = \frac{p}{q}$ [where p & q are co-prime integers and $q \neq 0$]

Squaring on both sides,

$$(\sqrt{5})^2 = \left(\frac{p}{q}\right)^2$$

$$5 = \frac{p^2}{q^2}$$

$$\boxed{p^2 = 5q^2} \longrightarrow ①$$

ie, p^2 is divisible by 5.

$\Rightarrow$ p is also divisible by 5. $\longrightarrow$ (2)

Let $p = 5m$.

Squaring on both sides,

$$p^2 = (5m)^2$$

$$5q^2 = 25m^2 \quad [\text{By } (1)]$$

$$q^2 = \frac{\cancel{25}^{5}}{\cancel{5}} m^2$$

$$\boxed{q^2 = 5m^2} \longrightarrow (3)$$

ie, q^2 is divisible by 5.

$\Rightarrow$ q is also divisible by 5. $\longrightarrow$ (4)

From (2) & (4),

p and q are divisible by 5.

ie, p & q have a common factor 5.

This is a contradiction to our assumption. ($\Rightarrow\Leftarrow$)

$\therefore$ Our assumption is wrong.

Thus, $\sqrt{5}$ is an irrational number.

Hence proved.

HOMEWORK

3) P.T $\sqrt{3}$ is an irrational number.

4) P.T $\sqrt{7}$ is an irrational number by contradiction method.

5) Show that $3\sqrt{3}$ is not a rational number.

Proof:

Let us assume $3\sqrt{3}$ is a rational number.

$$\therefore 3\sqrt{3} = \frac{p}{q}.$$

$$\sqrt{3} = \frac{p}{3q}.$$

Here, LHS $= \sqrt{3}$, is an irrational number

RHS $= \frac{p}{3q}$, is a rational number.

$\therefore$ LHS $\neq$ RHS.

Thus, $3\sqrt{3}$ is not a rational number.

Hence proved.

6) P.T $2+\sqrt{2}$ is an irrational number.

Proof:

Let us assume $2+\sqrt{2}$ is a rational number.

$$\therefore 2+\sqrt{2} = \frac{p}{q}$$

$$\sqrt{2} = \frac{p}{q} - 2$$

Here, LHS $= \sqrt{2}$, is an irrational number.

RHS $= \frac{p}{q} - 2$, is a rational number.

$\therefore$ LHS $\neq$ RHS.

Thus, $2 + \sqrt{2}$ is an irrational number.

Hence proved.

$\boxed{\text{HW}}$

7) PT. $\frac{3 - \sqrt{3}}{7}$ is not a rational number.

---

8) Show that $\sqrt{2} + \sqrt{3}$ is not a rational number.

Proof:

Let $\sqrt{2} + \sqrt{3} = a$ $\left[a = \frac{p}{q}\right.$, where p & q are co-prime integers & $q \neq 0$]

Squaring on both sides,

$$(\sqrt{2} + \sqrt{3})^2 = a^2$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

[$\because \sqrt{2} \times \sqrt{3} = \sqrt{6}$

Irr Irr Irr.

$$(\sqrt{2})^2 + (\sqrt{3})^2 + 2(\sqrt{2})(\sqrt{3}) = a^2$$

$$2 + 3 + 2\sqrt{6} = a^2$$

$$5 + 2\sqrt{6} = a^2$$

$$2\sqrt{6} = a^2 - 5$$

$$\sqrt{6} = \frac{a^2 - \sqrt{5}}{2}$$

Here, LHS $= \sqrt{6}$, is an irrational number.

and RHS $= \frac{(a^2-5)}{2}$ is a rational number

$\therefore$ LHS $\neq$ RHS.

This contradicts our assumption.

$\therefore$ $\sqrt{2}+\sqrt{3}$ is an irrational number.

Hence verified.

$\boxed{\text{HW}}$

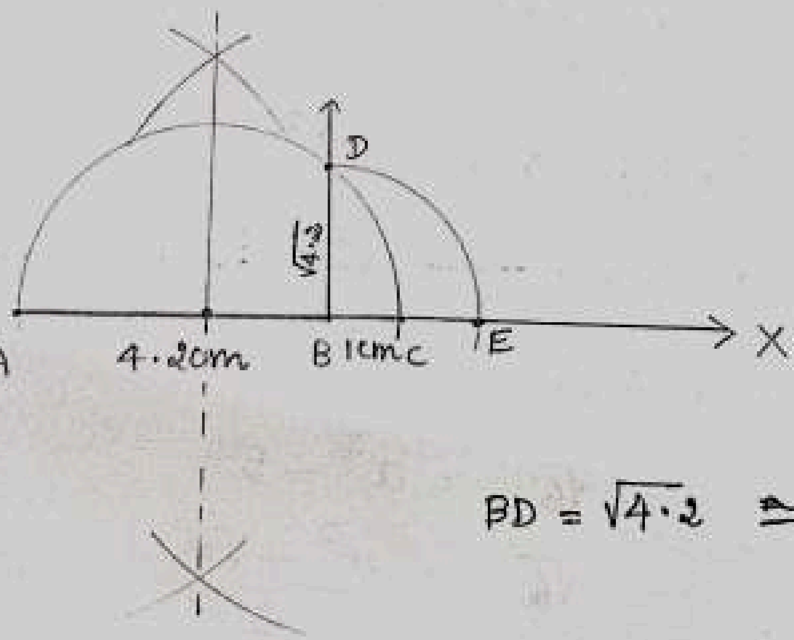
9) Prove that $\sqrt{3}+\sqrt{5}$ is not a rational number.

HW

9) Prove that $\sqrt{3}+\sqrt{5}$ is not a rational number.

GEOMETRICAL REPRESENTATION OF IRRATIONAL NUMBERS

1) Represent $\sqrt{4.2}$ geometrically.



$BD = \sqrt{4.2} \simeq 2.1$

CONSTRUCTION

(1) Draw $\overline{AB} = 4.2$ cm.

(2) Produce AB to point C s.t $\overline{BC} = 1$ cm

(3) Construct the midpoint 'O' of AC.

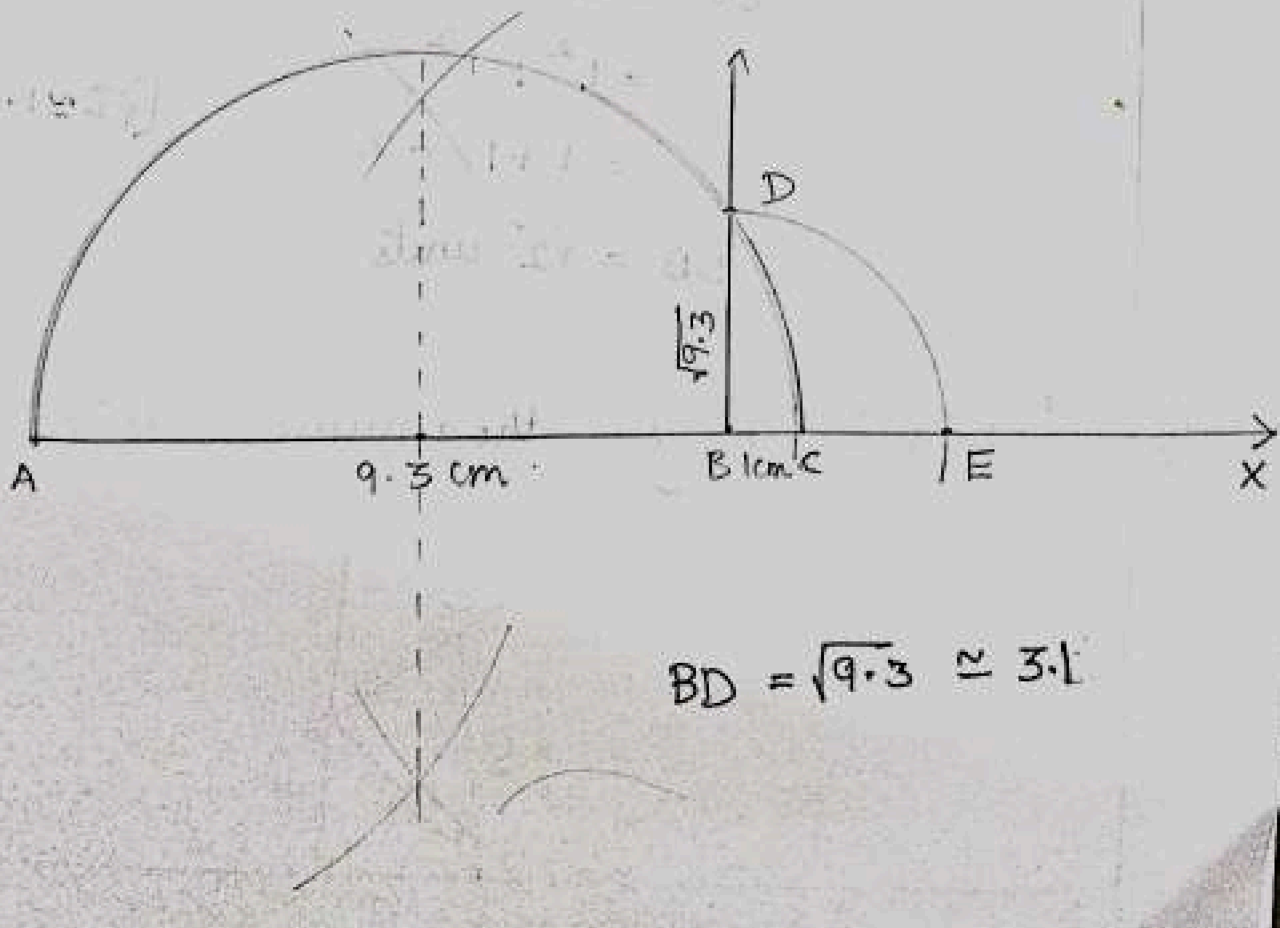
(4) With 'O' as centre and OA as radius draw a semicircle.

(5) Draw $BD \perp AC$ which cuts the semicircle at 'D'.

(6) With B as centre and BD as radius draw an arc which cuts AX at 'E'.

(7) Therefore $\overline{BD} = \overline{BE} = \sqrt{4.2}$.

2) Construct $\sqrt{9.3}$ geometrically.



$BD = \sqrt{9.3} \simeq 3.1$

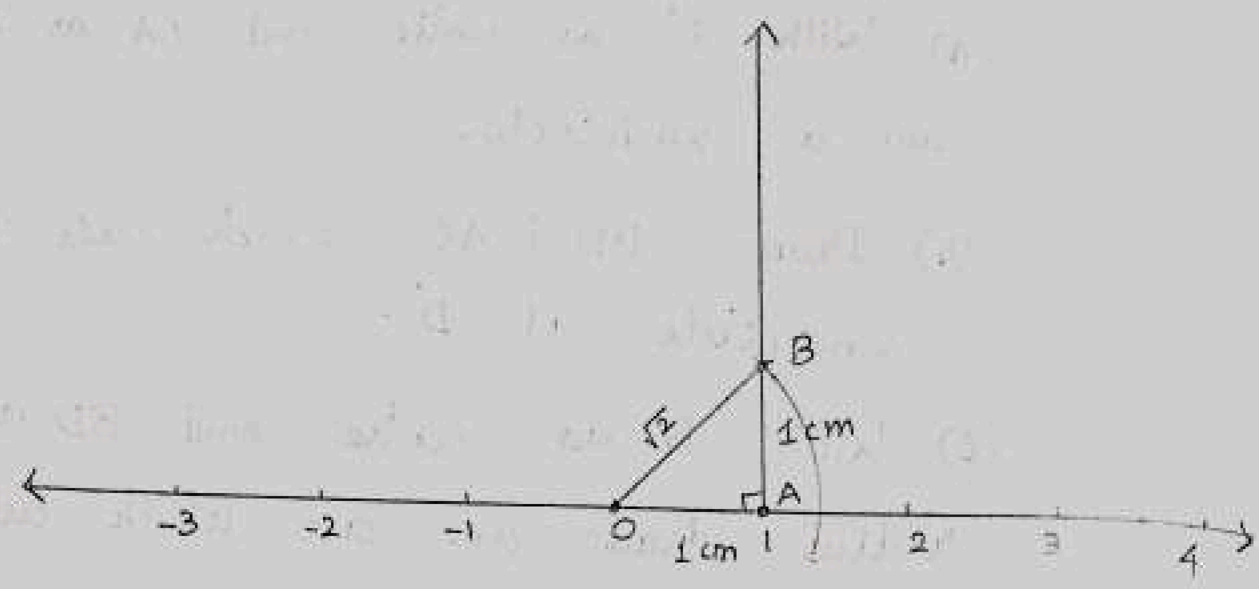
CONSTRUCTION

(HW)

REPRESENTING IRRATIONAL NUMBERS ON A NUMBER LINE

3) Represent $\sqrt{2}$ on a number line

[Spiral square root]



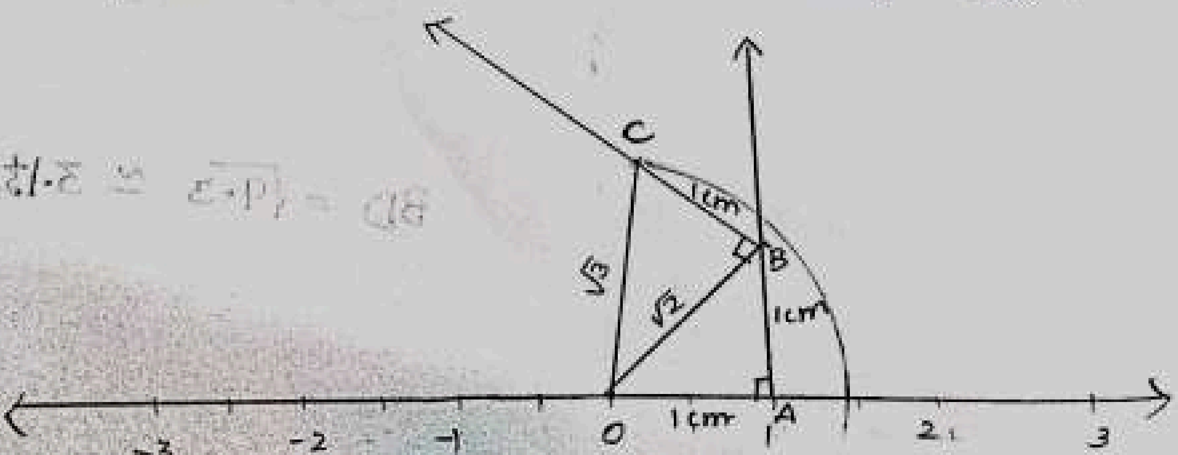
By pythagoras theorem,

In ΔOAB,

$$OB^2 = OA^2 + AB^2$$
$$= 1^2 + 1^2$$
$$= 1 + 1 = 2$$
$$OB = \sqrt{2} \text{ units}$$

$[\sqrt{2} \cong 1.414]$

4) Represent $\sqrt{3}$ on a number line.



By pythagoras thm,

In $\triangle OBC$,

$$OC^2 = OB^2 + BC^2$$
$$= (\sqrt{2})^2 + 1^2$$
$$= 2 + 1 = 3$$
$$OC = \sqrt{3} \text{ units}$$

$[\sqrt{3} \simeq 1.732.$

$\boxed{\text{HOMEWORK}}$

5) Represent $\sqrt{4}$ & $\sqrt{5}$ on a number line.

Ex 1.3 pg : 9.

9. Classify the following numbers as rational or irrational :

i) $\sqrt{23}$ = Irrational number.

ii) $\sqrt{225} = 15$ = Rational number.

iii) $0.3796 = \frac{3796}{10000}$ = Rational number.

iv) $7.478478\ldots = 7.\overline{478}$ = Rational number.

v) $1.101001000100001\ldots$ = Irrational number.

Laws of Radicals

1) $\sqrt[n]{a} = (a)^{1/n}$

2) $(\sqrt[n]{a})^n = \sqrt[n]{a^n} = a$

3) $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$

4) $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}}$

5) $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

I. Simplify:

a) $\sqrt[3]{4^3}$

soln:

$\sqrt[3]{4^3} = 4$ $\qquad [\because \sqrt[n]{a^n} = a$.

$\boxed{\text{Ans} = 4}$

b) $(\sqrt[3]{7})^3$

soln:

$(\sqrt[3]{7})^3 = \sqrt[3]{7^3}$

$\boxed{\text{Ans} = 7}$ $\qquad [\because \sqrt[n]{a^n} = a$

c) $\sqrt[4]{81}$

$$\begin{array}{r|l} 3 & 81 \\ 3 & 27 \\ 3 & 9 \\ & 3 \end{array}$$

soln:

$\sqrt[4]{81} = \sqrt[4]{3^4}$

$\boxed{\text{Ans} = 3}$ $\qquad [\because \sqrt[n]{a^n} = a$.

d) $\sqrt{\sqrt{625}}$

<u>soln:</u>

$$\sqrt[2]{\sqrt[2]{625}} = \sqrt[4]{625} \qquad \left[\because \sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}\right.$$

$$= \sqrt[4]{5^4}$$

$$\begin{array}{r|l} 5 & 625 \\ \hline 5 & 125 \\ \hline 5 & 25 \\ \hline & 5 \end{array}$$

$$\boxed{\text{Ans} = 5}$$

e) $\sqrt[3]{2} \cdot \sqrt[3]{32}$

<u>soln:</u>

$$\sqrt[3]{2} \cdot \sqrt[3]{32} = \sqrt[3]{2 \times 32}$$

$$\left[\sqrt[m]{a} \cdot \sqrt[m]{b} = \sqrt[n]{ab}\right.$$

$$= \sqrt[3]{64}$$

$$= \sqrt[3]{4^3}$$

$$\begin{array}{r|l} 4 & 64 \\ \hline 4 & 16 \\ \hline & 4 \end{array}$$

$$\boxed{\text{Ans} = 4}$$

f) $\sqrt[3]{216}$

<u>soln:</u>

$$\begin{array}{r|l} 6 & 216 \\ \hline 6 & 36 \\ \hline & 6 \end{array}$$

$$\sqrt[3]{216} = (216)^{1/3}$$

$$= (6^{\cancel{3}})^{1/\cancel{3}} = 6^1$$

$$\boxed{\text{Ans} = 6}$$

$$\begin{array}{r|l} 3 & 54 \\ \hline 3 & 18 \\ \hline 2 & 6 \\ \hline & 3 \end{array}$$

g) $\sqrt[3]{54}$

<u>soln:</u> $\sqrt[3]{54} = \sqrt[3]{3 \times 3 \times 3 \times 2}$

$$\boxed{\text{Ans} = 3\sqrt[3]{2}}$$

h) $\sqrt{5} \times \sqrt{10}$

soln:

$= \sqrt{5 \times 10}$

$= \sqrt{\underbrace{5 \times 5}_{} \times 2}$

$\boxed{\text{Ans} = 5\sqrt{2}}$

i) $6.\sqrt{9} \times 9.\sqrt{3}$

soln:

$6\sqrt{9} \times 9\sqrt{3} = (6 \times 9)\sqrt{9}.\sqrt{3}$

$= 54\sqrt{27}$

$= 54\sqrt{\underbrace{3 \times 3}_{} \times 3}$

$= (54 \times 3)\sqrt{3}$

$\boxed{\text{Ans} = 162\sqrt{3}}$

EXERCISE 1.5 (Pg: 24)

1. Classify the following numbers as rational or irrational.

(i) $2 - \sqrt{5}$ (ii) $(3+\sqrt{23}) - \sqrt{23}$ (iii) $\frac{2\sqrt{7}}{7\sqrt{7}}$

(iv) $\frac{1}{\sqrt{2}}$ (v) 2π.

Rational Numbers: $(3+\sqrt{23}) - \sqrt{23}$, $\frac{2\sqrt{7}}{7\sqrt{7}}$

Irrational Numbers: $2-\sqrt{5}$, $\frac{1}{\sqrt{2}}$, 2π

2. Simplify each of the following expressions

(i) $(3+\sqrt{3})(2+\sqrt{2})$

soln:

$$(3+\sqrt{3})(2+\sqrt{2}) = 6 + 3\sqrt{2} + 2\sqrt{3} + \sqrt{3}.\sqrt{2}$$

$$\text{Ans} = 6 + 3\sqrt{2} + 2\sqrt{3} + \sqrt{6}$$

(HOME WORK)

(ii) $(3+\sqrt{3})(3-\sqrt{3})$

(iii) $(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})$ [Hint: $(a-b)(a+b) = a^2-b^2$]

(iv) $(\sqrt{5}+\sqrt{2})^2$

[$(a+b)^2 = a^2+b^2+2ab$]

soln:

$$(\sqrt{5}+\sqrt{2})^2 = (\sqrt{5})^2+(\sqrt{2})^2+2(\sqrt{5})(\sqrt{2})$$

$$= 5+2+2\sqrt{10}$$

$$\text{Ans} = \underline{7+2\sqrt{10}}$$

[HOMEWORK]

(1) Classify the following numbers are rational or irrational numbers.

(a) $\sqrt{4}$ (b) $3\sqrt{2}$ (c) $\sqrt{\frac{9}{27}}$.

(d) $\sqrt{100}$.

(2) Simplify.

(a) $(\sqrt{3}+1)^2$

(b) $(\sqrt{5}+1)(\sqrt{5}-1)$

EXERCISE 1.6 (P:26)

1. Find :-

(i) $64^{1/2}$

soln:

$$64^{1/2} = \sqrt{64} \quad [(a)^{1/n} = \sqrt[n]{a}]$$

$$= \sqrt{8 \times 8}$$

Ans = 8

$64^{1/2} = (8^2)^{1/2}$
Ans = 8

(ii) $32^{1/5}$

soln:

$$32^{1/5} = \sqrt[5]{32}$$

$$= \sqrt[5]{2 \times 2 \times 2 \times 2 \times 2}$$

Ans = 2

$(2^5)^{1/5} = 2.$

(iii) $125^{1/3}$

soln:

$$125^{1/3} = \sqrt[3]{125}$$

$$= \sqrt[3]{5 \times 5 \times 5}$$

Ans = 5

2. Find:-

(i) $9^{3/2}$

soln:

$$9^{3/2} = (3^2)^{3/2} \quad [\because (a^m)^n = a^{mn}]$$

$$= 3^3$$

Ans = 27.

(ii) $(32)^{2/5}$

soln:-

$$(32)^{2/5} = (2^{\cancel{5}})^{2/\cancel{5}}$$

$$= 2^2$$

$$Ans = 4$$

(iii) $(125)^{-1/3}$

soln:-

$$(125)^{-1/3} = \frac{1}{(125)^{1/3}} \qquad \left[a^{-n} = \frac{1}{a^n} \right.$$

$$= \frac{1}{(5^{\cancel{3}})^{\frac{1}{\cancel{3}}}}$$

$$Ans = \frac{1}{5}$$

HW

(iv) $16^{3/4}$.

3. Simplify :-

(i) $2^{2/3} \cdot 2^{1/5}$.

soln: $2^{2/3} \cdot 2^{1/5} = 2^{2/3 + \frac{1}{5}} \qquad \left[\because a^m \cdot a^n = a^{m+n} \right.$

$$= 2^{\frac{10+3}{15}}$$

$$Ans = 2^{\frac{13}{15}}$$

(ii) $\left(\frac{1}{3^3}\right)^7$

soln:-

$$\left(\frac{1}{3^3}\right)^7 = \frac{1^7}{(3^3)^7} \qquad \left[\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}\right.$$

$$\text{Ans} = \frac{1}{3^{21}} \qquad \left[(a^m)^n = a^{mn}\right.$$

(iii) $\frac{11^{1/2}}{11^{1/4}}$

soln:-

$$\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}} = 11^{\frac{1}{2}-\frac{1}{4}} \qquad \left[\frac{a^m}{a^n} = a^{m-n}\right.$$

$$= 11^{\frac{2-1}{4}}$$

$$\text{Ans} = 11^{\frac{1}{4}}$$

(iv) $7^{\frac{1}{2}} \cdot 8^{\frac{1}{2}}$

soln:-

$$7^{\frac{1}{2}} \cdot 8^{\frac{1}{2}} = (7 \times 8)^{\frac{1}{2}} \qquad \left[\because a^m \times b^m = (ab)^m\right.$$

$$\text{Ans} = (56)^{\frac{1}{2}}$$

$$\begin{array}{r|l} 2 & 56 \\ 2 & 28 \\ 2 & 14 \\ & 7 \end{array}$$

$\boxed{\text{HW}}$

(v) $16^{\frac{1}{3}} \cdot 4^{\frac{1}{3}}$.

Operations on Irrational Numbers

1. Simplify :-

1) $4\sqrt{2} + 3\sqrt{32}$.

soln:-

$$4\sqrt{2} + 3\sqrt{32} = 4\sqrt{2} + 3\sqrt{\underbrace{2\times2}\times\underbrace{2\times2}\times 2}$$

$$= 4\sqrt{2} + (3\times2\times2)\sqrt{2}$$

$$= 4\sqrt{2} + 12\sqrt{2}$$

$$\text{Ans} = 16\sqrt{2}$$

2) $8\sqrt{3} - 2\sqrt{3} + 4\sqrt{3}$

soln:-

$$8\sqrt{3} - 2\sqrt{3} + 4\sqrt{3} = (8-2+4)\sqrt{3}$$

$$\text{Ans} = 10\sqrt{3}$$

3) $\sqrt{48} - \sqrt{72} - \sqrt{27} + 2\sqrt{18}$

2 | 48
2 | 24
2 | 12
2 | 6
3

2 | 72
2 | 36
2 | 18
3 | 9
3

soln:-

$$\sqrt{48} = \sqrt{\underbrace{2\times2}\times\underbrace{2\times2}\times3}$$

$$= 2\times2\sqrt{3}$$

$$= 4\sqrt{3}$$

$$\sqrt{72} = \sqrt{\underbrace{2\times2}\times2\times\underbrace{3\times3}}$$

$$= 2\times3\sqrt{2}$$

$$= 6\sqrt{2}$$

$$\sqrt{27} = \sqrt{3\times3\times3}$$
$$= 3\sqrt{3}.$$

$$2\sqrt{18} = 2\sqrt{2\times\underline{3\times3}}$$
$$= (2\times3)\sqrt{2}$$
$$= 6\sqrt{2}.$$

$$\therefore \sqrt{48}-\sqrt{72}-\sqrt{27}+2\sqrt{18} = 4\sqrt{3}-\cancel{6\sqrt{2}}-3\sqrt{3}+\cancel{6\sqrt{2}}$$
$$= 4\sqrt{3}-3\sqrt{3}$$
$$\text{Ans} = \sqrt{3}$$

4) Divide $4\sqrt{28}$ by $3\sqrt{7}$

soln:-

$$\frac{4\sqrt{28}}{3\sqrt{7}} = \frac{4}{3}\sqrt{\frac{\cancel{28}\,4}{\cancel{7}}}$$
$$= \frac{4}{3}\sqrt{4}$$
$$= \frac{4}{3}\times 2$$
$$\text{Ans} = \frac{8}{3}.$$

5 $\sqrt{120}\times\sqrt{45}$

$$\begin{array}{r|l} 2 & 120 \\ 2 & 60 \\ 2 & 30 \\ 3 & 15 \\ & 5 \end{array} \qquad \begin{array}{r|l} 5 & 45 \\ 3 & 9 \\ & 3 \end{array}$$

soln:

$$\sqrt{120}\times\sqrt{45} = \sqrt{\underline{2\times2}\times2\times3\times5}\times\sqrt{\underline{3\times3}\times5}$$
$$= 2\sqrt{30}\times3\sqrt{5}$$
$$= 6\sqrt{30\times5}$$
$$= 6\sqrt{150}$$

$$= 6\sqrt{2 \times 3 \times 5 \times 5}$$

$$\begin{array}{r|l} 5 & 150 \\ \hline 3 & 30 \\ \hline 2 & 10 \\ \hline & 5 \end{array}$$

$$= 6 \times 5\sqrt{6}$$

Ans $= 30\sqrt{6}$

HOMEWORK

6) $$\frac{\sqrt{3} \times \sqrt{8} \times \sqrt{39}}{\sqrt{24} \times \sqrt{26}}$$

7) $\sqrt{5} + \sqrt{20} + \sqrt{45}$

8) $(8 + 5\sqrt{3})^2$

COMPARISON OF IRRATIONAL NUMBERS

1) Which is greater?

(a) $\sqrt[3]{7}$ or $\sqrt[4]{8}$

soln:

LCM of 3 and 4 = 12.

$$\sqrt[3]{7} = \sqrt[3 \times 4]{7^4} = \sqrt[12]{2401}$$

$$\sqrt[4]{8} = \sqrt[4 \times 3]{8^3} = \sqrt[12]{512}$$

$$\sqrt[12]{2401} > \sqrt[12]{512}$$

$$\therefore \sqrt[3]{7} > \sqrt[4]{8}$$

(b) $\sqrt[3]{12}$ or $\sqrt[6]{6}$

Soln:

LCM of 3 and 6 = 6

$\sqrt[3]{12} = \sqrt[3\times 2]{12^2} = \sqrt[6]{144}$

$\sqrt[6]{6} = \sqrt[6\times 1]{6^1} = \sqrt[6]{6}$

$\sqrt[6]{144} > \sqrt[6]{6}$

$\therefore \sqrt[3]{12} > \sqrt[6]{6}$

HOMEWORK

(c) $\sqrt{3}$ or $\sqrt[4]{7}$

(d) $\sqrt[3]{4}$ or $\sqrt[4]{9}$

2> Rewrite in ascending order

(a) $\sqrt[6]{6}$, $\sqrt[3]{7}$, $\sqrt[4]{8}$

Soln:-

LCM of 3 , 4 and 6 = 12

$\sqrt[6]{6} = \sqrt[6\times 2]{6^2} = \sqrt[12]{36}$

$\sqrt[3]{7} = \sqrt[3\times 4]{7^4} = \sqrt[12]{2401}$

$\sqrt[4]{8} = \sqrt[4\times 3]{8^3} = \sqrt[12]{512}$

$\therefore$ Ascending order is $\sqrt[12]{36}$, $\sqrt[12]{512}$, $\sqrt[12]{2401}$.

HOMEWORK

(b) Rearrange in descending order.

$\sqrt[4]{9}$, $\sqrt[6]{26}$, $\sqrt[3]{5}$.

RATIONALISING THE DENOMINATOR

1) Rationalise the denominators

i) $\frac{1}{\sqrt{7}}$.

soln:- $[\because \sqrt{7} \times \sqrt{7} = 7$

$$\frac{1}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{7}}{7}$$

$$\text{Ans} = \frac{\sqrt{7}}{7}.$$

2) $\frac{1}{\sqrt{2}+\sqrt{3}}$

soln:- Conjugate of $\sqrt{2}+\sqrt{3} = \sqrt{2}-\sqrt{3}$

$$\frac{1}{\sqrt{2}+\sqrt{3}} \times \frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}} = \frac{(\sqrt{2}-\sqrt{3})}{\underbrace{(\sqrt{2}+\sqrt{3})}_{(a+b)}\underbrace{(\sqrt{2}-\sqrt{3})}_{(a-b)}} \quad = a^2-b^2$$

$$= \frac{\sqrt{2}-\sqrt{3}}{(\sqrt{2})^2-(\sqrt{3})^2}$$

$$= \frac{\sqrt{2}-\sqrt{3}}{2-3}$$

$$= \frac{\sqrt{2}-\sqrt{3}}{-1}$$

$$\text{Ans} = \sqrt{3}-\sqrt{2}$$

3) $\frac{5}{2\sqrt{5}}$.

soln:-

$$\frac{5}{2\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\cancel{5}\sqrt{5}}{2\times\cancel{5}}$$

$$\text{Ans} = \frac{\sqrt{5}}{2}$$

HOMEWORK

4) $\frac{3}{4\sqrt{3}}$

5) $\frac{2}{\sqrt{6}-\sqrt{5}}$

6) $\frac{1}{\sqrt{12}}$

7) $\frac{30}{5\sqrt{3}-3\sqrt{5}}$

soln:- Conjugate of $5\sqrt{3}-3\sqrt{5}$ is $5\sqrt{3}+3\sqrt{5}$

$$\frac{30}{5\sqrt{3}-3\sqrt{5}} \times \frac{5\sqrt{3}+3\sqrt{5}}{5\sqrt{3}+3\sqrt{5}} = \frac{30\,(5\sqrt{3}+3\sqrt{5})}{(5\sqrt{3})^2-(3\sqrt{5})^2}$$

$$= \frac{150\sqrt{3}+90\sqrt{5}}{(25\times3)-(9\times5)}$$

$$= \frac{150\sqrt{3}+90\sqrt{5}}{75-45}$$

$$= \frac{150\sqrt{3}+90\sqrt{5}}{30}$$

$$= \frac{\cancel{30}(5\sqrt{3} + 3\sqrt{5})}{\cancel{30}}$$

$$Ans = 5\sqrt{3} + 3\sqrt{5}$$

8) $\dfrac{2\sqrt{3} + 5\sqrt{7}}{\sqrt{5} + \sqrt{6}}$

soln :

Conjugate of $\sqrt{5} + \sqrt{6}$ is $\sqrt{5} - \sqrt{6}$.

$$\frac{2\sqrt{3} + 5\sqrt{7}}{\sqrt{5} + \sqrt{6}} \times \frac{\sqrt{5} - \sqrt{6}}{\sqrt{5} - \sqrt{6}} = \frac{(2\sqrt{3} + 5\sqrt{7})(\sqrt{5} - \sqrt{6})}{(\sqrt{5})^2 - (\sqrt{6})^2}$$

$$= \frac{2\sqrt{15} - 2\sqrt{18} + 5\sqrt{35} - 5\sqrt{42}}{5 - 6}$$

$$= \frac{2\sqrt{15} - 2\sqrt{18} + 5\sqrt{35} - 5\sqrt{42}}{-1}$$

$$= -2\sqrt{15} + 2\sqrt{9 \times 2} - 5\sqrt{35} + 5\sqrt{42}$$

$$Ans = -2\sqrt{15} + 6\sqrt{2} - 5\sqrt{35} + 5\sqrt{42}.$$

HOMEWORK

9) $\dfrac{1}{\sqrt{6} - \sqrt{7}}$.

10) $\dfrac{6 - 4\sqrt{2}}{6 + 4\sqrt{2}}$

11) $\dfrac{5 + \sqrt{3}}{5 - \sqrt{3}}$.

12) $\dfrac{4}{2+\sqrt{3}+\sqrt{7}}$.

soln.

Conjugate of $(2+\sqrt{3})+\sqrt{7}$ is $(2+\sqrt{3})-\sqrt{7}$.

$$\frac{4}{(2+\sqrt{3})+\sqrt{7}} \times \frac{(2+\sqrt{3})-\sqrt{7}}{(2+\sqrt{3})-\sqrt{7}} = \frac{4\left[(2+\sqrt{3})-\sqrt{7}\right]}{(2+\sqrt{3})^2-(\sqrt{7})^2}$$

$$= \frac{4\left[(2+\sqrt{3})-\sqrt{7}\right]}{2^2+(\sqrt{3})^2+4\sqrt{3}-7}$$

$$= \frac{4\left[(2+\sqrt{3})-\sqrt{7}\right]}{\cancel{4}+\cancel{3}+4\sqrt{3}\cancel{-7}}$$

$$= \frac{\cancel{4}\left[(2+\sqrt{3})-\sqrt{7}\right]}{\cancel{4}\sqrt{3}}$$

$$= \frac{(2+\sqrt{3})-\sqrt{7}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{\sqrt{3}\left[(2+\sqrt{3})-\sqrt{7}\right]}{3}$$

$$\text{Ans} = \frac{2\sqrt{3}+3-\sqrt{21}}{3}$$

13) $\dfrac{1}{3+\sqrt{5}-2\sqrt{2}}$

<u>soln</u>:

Conjugate of $(3+\sqrt{5})-2\sqrt{2}$ is $(3+\sqrt{5})+2\sqrt{2}$.

$$\frac{1}{(3+\sqrt{5})-2\sqrt{2}} \times \frac{(3+\sqrt{5})+2\sqrt{2}}{(3+\sqrt{5})+2\sqrt{2}} = \frac{(3+\sqrt{5})+2\sqrt{2}}{(3+\sqrt{5})^2-(2\sqrt{2})^2}$$

$$= \frac{(3+\sqrt{5})+2\sqrt{2}}{3^2+(\sqrt{5})^2+6\sqrt{5}-(4\times 2)}$$

$$= \frac{3+\sqrt{5}+2\sqrt{2}}{9+5+6\sqrt{5}-8}$$

$$= \frac{3+\sqrt{5}+2\sqrt{2}}{6+6\sqrt{5}} \times \frac{6-6\sqrt{5}}{6-6\sqrt{5}}$$

$$= \frac{(3+\sqrt{5}+2\sqrt{2})\ (6-6\sqrt{5})}{(6)^2-(6\sqrt{5})^2} \quad (36\times 5 = 180)$$

$$= \frac{18-18\sqrt{5}+6\sqrt{5}-30+12\sqrt{2}-12\sqrt{10}}{36-180}$$

$$= \frac{-12-12\sqrt{5}+12\sqrt{2}-12\sqrt{10}}{-144}$$

$$= \frac{\cancel{+12}\,[1+\sqrt{5}-\sqrt{2}+\sqrt{10}]}{\cancel{+144}\ 12}$$

$$\text{Ans} = \frac{1+\sqrt{5}-\sqrt{2}+\sqrt{10}}{12}$$

HOMEWORK

14) $\dfrac{1}{(\sqrt{3}+\sqrt{2})+\sqrt{5}}$

(2) Rationalise the denominator and simplify:-

(i) $\dfrac{\sqrt{5}-2}{\sqrt{5}+2} - \dfrac{\sqrt{5}+2}{\sqrt{5}-2}$.

soln:-

Step: 1

$$\frac{\sqrt{5}-2}{\sqrt{5}+2} \times \frac{\sqrt{5}-2}{\sqrt{5}-2} = \frac{(\sqrt{5}-2)^2}{(\sqrt{5})^2-(2)^2}$$

$$= \frac{(\sqrt{5})^2+(2)^2-2(\sqrt{5})(2)}{5-4}$$

$$= \frac{5+4-4\sqrt{5}}{1}$$

$$= 9-4\sqrt{5}$$

Step: 2

$$\frac{\sqrt{5}+2}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2} = \frac{(\sqrt{5}+2)^2}{(\sqrt{5})^2-(2)^2}$$

$$= \frac{(\sqrt{5})^2+(2)^2+2(\sqrt{5})(2)}{5-4}$$

$$= \frac{5+4+4\sqrt{5}}{}$$

$= 9 + 4\sqrt{5}$

Step: 3

$$\frac{\sqrt{5}-2}{\sqrt{5}+2} - \frac{\sqrt{5}+2}{\sqrt{5}-2} = (9-4\sqrt{5}) - (9+4\sqrt{5})$$

$$= \cancel{9} - 4\sqrt{5} - \cancel{9} - 4\sqrt{5}$$

$$\text{Ans} = -8\sqrt{5}$$

HOMEWORK

(ii) $\dfrac{2+\sqrt{3}}{2-\sqrt{3}} - \dfrac{2-\sqrt{3}}{2+\sqrt{3}}$

Show that,

(iii) $\dfrac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} - \dfrac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} - \dfrac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}} = 1$

Proof:-

$\boxed{\text{Step: 1}}$

$$\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} \times \frac{\sqrt{10}-\sqrt{3}}{\sqrt{10}-\sqrt{3}} = \frac{7\sqrt{3}(\sqrt{10}-\sqrt{3})}{(\sqrt{10})^2 - (\sqrt{3})^2}$$

$$= \frac{7\sqrt{3}\,(\sqrt{10}-\sqrt{3})}{10-3}$$

$$= \frac{\cancel{7}\sqrt{3}\,(\sqrt{10}-\sqrt{3})}{\cancel{7}}$$

$$= \sqrt{30} - 3 \longrightarrow ①$$

$\boxed{\text{Step:2}}$

$$\frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} \times \frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}-\sqrt{5}} = \frac{2\sqrt{5}(\sqrt{6}-\sqrt{5})}{(\sqrt{6})^2-(\sqrt{5})^2}$$

$$= \frac{2\sqrt{30}-10}{6-5} \quad (2\times5)$$

$$= 2\sqrt{30}-10 \longrightarrow (2)$$

$\boxed{\text{Step:3}}$

$$\frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}} \times \frac{\sqrt{15}-3\sqrt{2}}{\sqrt{15}-3\sqrt{2}} = \frac{3\sqrt{2}(\sqrt{15}-3\sqrt{2})}{(\sqrt{15})^2-(3\sqrt{2})^2}$$

$[9\times2=18]$

$$= \frac{3\sqrt{2}(\sqrt{15}-3\sqrt{2})}{15-18}$$

$$= \frac{\cancel{3}\sqrt{2}(\sqrt{15}-3\sqrt{2})}{-\cancel{3}}$$

$$= \frac{\sqrt{30}-6}{-1} \quad [\sqrt{2}\times3\sqrt{2}=3\times2=6$$

$$= -(\sqrt{30}-6)$$

$$= 6-\sqrt{30} \longrightarrow (3)$$

From (1), (2) & (3)

$$= \sqrt{30}-3-(2\sqrt{30}-10)-(6-\sqrt{30})$$

$$= \cancel{\sqrt{30}}-3-\cancel{2\sqrt{30}}+10-6+\cancel{\sqrt{30}}$$

$$= 10-9$$

$$= 1$$

Hence proved.

⟨3⟩ Find the values of a and b

(1) $$\frac{\sqrt{3}-1}{\sqrt{3}+1} = a + b\sqrt{3}.$$

soln:-

LHS :

$$\frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{(\sqrt{3}-1)^2}{(\sqrt{3})^2-(1)^2}$$

$$= \frac{(\sqrt{3})^2+(1)^2-2(\sqrt{3})(1)}{3-1}$$

$$= \frac{3+1-2\sqrt{3}}{2}$$

$$= \frac{4-2\sqrt{3}}{2}$$

$$= \frac{\cancel{2}(2-\sqrt{3})}{\cancel{2}}$$

$$= 2-\sqrt{3}.$$

$\therefore\ 2-1.\sqrt{3} = a+b\sqrt{3}$

$\Rightarrow\ 2+(-1)\cdot\sqrt{3} = a+b\cdot\sqrt{3}.$

Here $a=2$, $b=-1$.

(ii) $\dfrac{5+2\sqrt{3}}{7+4\sqrt{3}} = a+b\sqrt{3}$.

soln:

LHS :

$$\frac{5+2\sqrt{3}}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}} = \frac{(5+2\sqrt{3})(7-4\sqrt{3})}{(7)^2-(4\sqrt{3})^2}$$

$(16\times3 \quad 48)$

$$= \frac{(5+2\sqrt{3})(7-4\sqrt{3})}{49-48}$$

$$= \frac{35-20\sqrt{3}+14\sqrt{3}-24}{1}$$

$$= 11-6\sqrt{3}.$$

$\therefore 11-6\sqrt{3} = a+b\sqrt{3}$

$a=11$

$b=-6$.

(iii) $\dfrac{\sqrt{7}-1}{\sqrt{7}+1} - \dfrac{\sqrt{7}+1}{\sqrt{7}-1} = a+b\sqrt{7}$

soln:-

$\boxed{\text{Step: 1}}$

$$\frac{\sqrt{7}-1}{\sqrt{7}+1} \times \frac{\sqrt{7}-1}{\sqrt{7}-1} = \frac{(\sqrt{7}-1)^2}{(\sqrt{7})^2-(1)^2}$$

$$= \frac{(\sqrt{7})^2+(1)^2-2(\sqrt{7})(1)}{7-1}$$

$$= \frac{7+1-2\sqrt{7}}{6}$$

$$= \frac{8-2\sqrt{7}}{6}$$

$$= \frac{\cancel{2}(4-\sqrt{7})}{\cancel{6}\,3}$$

$$= \frac{4-\sqrt{7}}{3}.$$

$\boxed{\text{step: 2}}$

$$\frac{\sqrt{7}+1}{\sqrt{7}-1} \times \frac{\sqrt{7}+1}{\sqrt{7}+1} = \frac{(\sqrt{7}+1)^2}{(\sqrt{7})^2-(1)^2}$$

$$= \frac{(\sqrt{7})^2+(1)^2+2(\sqrt{7})(1)}{7-1}$$

$$= \frac{7+1+2\sqrt{7}}{6}$$

$$= \frac{8+2\sqrt{7}}{6}$$

$$= \frac{\cancel{2}(4+\sqrt{7})}{\cancel{6}\,3}$$

$$= \frac{4+\sqrt{7}}{3}$$

$\boxed{\text{step: 3}}$

$$\frac{4-\sqrt{7}}{3} - \left(\frac{4+\sqrt{7}}{3}\right) = \frac{\cancel{4}-\sqrt{7}-\cancel{4}-\sqrt{7}}{3}$$

$$= \frac{-2\sqrt{7}}{3}$$

$$= 0+\left(\frac{-2}{3}\right)\sqrt{7}$$

$$\therefore 0 + \left(\frac{-2}{3}\right)\sqrt{7} = a + b\sqrt{7}$$

Here $a = 0$, $b = \frac{-2}{3}$.

HW

(iv) $\frac{3+\sqrt{2}}{3-\sqrt{2}} = a + b\sqrt{2}$.

(v) $\frac{5+\sqrt{6}}{5-\sqrt{6}} = a + b\sqrt{6}$.

NOTES

$$(a+b)^2 = a^2 + b^2 + 2ab.$$

$$\Rightarrow \boxed{a^2 + b^2 = (a+b)^2 - 2ab}$$

Let $a = x$; $b = \frac{1}{x}$.

$$\left(x + \frac{1}{x}\right)^2 = x^2 + \left(\frac{1}{x}\right)^2 + 2 \times \cancel{x} \times \frac{1}{\cancel{x}}$$

$$\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2.$$

$$\Rightarrow \boxed{x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2}$$

1) If $x = 2 + \sqrt{3}$, find the value of

(a) $x + \frac{1}{x}$ (b) $x^2 + \frac{1}{x^2}$.

Soln:

(a) $x + \frac{1}{x}$.

given $x = 2+\sqrt{3}$.

$$\frac{1}{x} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$= \frac{2-\sqrt{3}}{(2)^2 - (\sqrt{3})^2}$$

$$= \frac{2-\sqrt{3}}{4-3}$$

$$\frac{1}{x} = 2-\sqrt{3}.$$

$$\therefore\ x+\frac{1}{x} = 2+\cancel{\sqrt{3}} + 2 - \cancel{\sqrt{3}}$$

$\boxed{\text{Ans} = 4}$

(b) $x^2 + \frac{1}{x^2}$.

We know that $x^2+\frac{1}{x^2} = \left(x+\frac{1}{x}\right)^2 - 2$

$$= 4^2 - 2$$

$$= 16 - 2$$

$\boxed{\text{Ans} = 14}$

Another method :-

$$x^2+\frac{1}{x^2} = (2+\sqrt{3})^2 + (2-\sqrt{3})^2$$

$$= 4+3+2(2)(\sqrt{3}) + 4+3-2(2)(\sqrt{3})$$

$$= 7 + 4\cancel{\sqrt{3}} + 7 - 4\cancel{\sqrt{3}}$$

$\boxed{\text{Ans} = 14}$

2) If $x = \frac{5-\sqrt{21}}{2}$, find the value of

(a) $x + \frac{1}{x}$ (b) $x^2 + \frac{1}{x^2}$

Soln:-

(a) $x + \frac{1}{x}$

Given $x = \frac{5-\sqrt{21}}{2}$

$\therefore \frac{1}{x} = \frac{2}{5-\sqrt{21}}$

$= \frac{2}{5-\sqrt{21}} \times \frac{5+\sqrt{21}}{5+\sqrt{21}}$

$= \frac{2(5+\sqrt{21})}{(5)^2-(\sqrt{21})^2}$

$= \frac{2(5+\sqrt{21})}{25-21}$

$= \frac{\cancel{2}(5+\sqrt{21})}{\cancel{4}\,2}$

$\frac{1}{x} = \frac{5+\sqrt{21}}{2}$

$\therefore x+\frac{1}{x} = \frac{5-\sqrt{21}}{2} + \frac{5+\sqrt{21}}{2}$

$= \frac{5-\cancel{\sqrt{21}}+5+\cancel{\sqrt{21}}}{2}$

$= \frac{10}{2}$

$\boxed{\text{Ans} = 5}$

(b) $x^2 + \frac{1}{x^2}$

WKT $x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2$

$= (5)^2 - 2$

$= 25 - 2$

$\boxed{\text{Ans} = 23}$

3) If $x = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$, find the value of $x^2 + \frac{1}{x^2}$.

soln:

Given $x = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$= \frac{(\sqrt{5}+\sqrt{3})^2}{(\sqrt{5})^2 - (\sqrt{3})^2}$

$= \frac{(\sqrt{5})^2 + (\sqrt{3})^2 + 2\sqrt{15}}{5-3}$

$= \frac{5+3+2\sqrt{15}}{2}$

$= \frac{8+2\sqrt{15}}{2}$

$= \frac{\cancel{2}(4+\sqrt{15})}{\cancel{2}}$

$x = 4 + \sqrt{15}$

$$\frac{1}{x} = \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} \times \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}$$

$$= \frac{(\sqrt{5}-\sqrt{3})^2}{(\sqrt{5})^2-(\sqrt{3})^2}$$

$$= \frac{(\sqrt{5})^2+(\sqrt{3})^2-2(\sqrt{5})(\sqrt{3})}{5-3}$$

$$= \frac{5+3-2\sqrt{15}}{2}$$

$$= \frac{8-2\sqrt{15}}{2}$$

$$= \frac{\cancel{2}(4-\sqrt{15})}{\cancel{2}}$$

$$\frac{1}{x} = 4-\sqrt{15}$$

$$\therefore x^2+\frac{1}{x^2} = \left(x+\frac{1}{x}\right)^2-2$$

$$= (4+\cancel{\sqrt{15}}+4-\cancel{\sqrt{15}})^2-2$$

$$= (8)^2-2$$

$$= 64-2$$

$$\boxed{\text{Ans} = 62}$$

HW

4) If $x = \sqrt{2} + 1$, find the value of

(a) $x + \frac{1}{x}$ (b) $x^2 + \frac{1}{x^2}$.

NOTES.

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a+b)^3 = a^3 + 3ab(a+b) + b^3$$

$$\Rightarrow a^3 + b^3 = (a+b)^3 - 3ab(a+b)$$

Let $a = x$; $b = \frac{1}{x}$.

$$x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^3 - 3 \times \cancel{x} \times \frac{1}{\cancel{x}}\left(x + \frac{1}{x}\right)$$

$$x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right)$$

Similarly,

$$x^3 - \frac{1}{x^3} = \left(x - \frac{1}{x}\right)^3 + 3\left(x - \frac{1}{x}\right)$$

1) If $x = 4 + \sqrt{15}$, find $x^3 - \frac{1}{x^3}$.

soln:

Given $x = 4 + \sqrt{15}$

$$\therefore \frac{1}{x} = \frac{1}{4+\sqrt{15}} \times \frac{4-\sqrt{15}}{4-\sqrt{15}}$$

$$= \frac{4-\sqrt{15}}{(4)^2 - (\sqrt{15})^2} = \frac{4-\sqrt{15}}{16-15}$$

$$\frac{1}{x} = 4 - \sqrt{15}$$

$$\therefore x - \frac{1}{x} = 4 + \sqrt{15} - (4 - \sqrt{15})$$

$= \cancel{4} + \sqrt{15} - \cancel{4} + \sqrt{15}$

$x - \frac{1}{x} = 2\sqrt{15}$

$\therefore x^3 - \frac{1}{x^3} = \left(x - \frac{1}{x}\right)^3 + 3\left(x - \frac{1}{x}\right)$

$= \left(2\sqrt{15}\right)^3 + 3\left(2\sqrt{15}\right)$

$= 8 \times 15\sqrt{15} + 6\sqrt{15}$ $\left[\underbrace{\sqrt{15}\cdot\sqrt{15}}\cdot\sqrt{15} = 15\sqrt{15}\right.$

$= 120\sqrt{15} + 6\sqrt{15}$

$x^3 - \frac{1}{x^3} = 126\sqrt{15}$

HW

2) If $x = 2 + \sqrt{3}$, find the value of

$x^3 + \frac{1}{x^3}$