A polynomial $p(x)$ in one variable x is of the form,

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x^1 + a_0$$

where $a_0, a_1, a_2 \ldots a_n$ are constants and $a_n \neq 0$.

Zeroes of a polynomial

A zero of a polynomial $p(x)$ is a number 'c' s.t $p(c) = 0$.

For eg,

$$p(x) = x - 1$$

Put $x = 1$,

$$\therefore p(1) = 1 - 1 = 0$$

$$\therefore p(1) = 0$$

$\therefore$ 1 is a zero of the polynomial $p(x)$

EXERCISE 2.2

1) Find the value of the polynomial $5x - 4x^2 + 3$ at,

(i) $x = 0$,

soln:- $p(x) = 5x - 4x^2 + 3$

$p(0) = 5(0) - 4(0)^2 + 3$

$= 0 - 0 + 3$

$\boxed{p(0) = 3}$

(ii) $x = (-1)$

soln:-

$p(x) = 5x - 4x^2 + 3$

$p(-1) = 5(-1) - 4(-1)^2 + 3$

$[-1 \times -1 = 1]$

$= -5 - 4 + 3$

$= -9 + 3$

$\boxed{p(-1) = -6}$

HW

(iii) $x = 2$.

2) Find $p(0)$, $p(1)$ & $p(2)$ for each of the following polynomials.

(i) $p(y) = y^2 - y + 1$

soln:-

$p(0) = (0)^2 - (0) + 1$

$\boxed{p(0) = 1}$

$$p(1) = (1)^2 - 1 + 1$$
$$= \cancel{1} - \cancel{1} + 1$$
$$\boxed{p(1) = 1}$$

$$p(2) = (2)^2 - 2 + 1$$
$$= 4 - 2 + 1$$
$$\boxed{p(2) = 3}$$

(ii) $p(t) = 2 + t + 2t^2 - t^3$

soln:-
$$p(0) = 2 + 0 + 2(0)^2 - (0)^3$$
$$= 2 + 0 + 0 - 0$$
$$\boxed{p(0) = 2}$$

$$p(1) = 2 + (1) + 2(1)^2 - (1)^3$$
$$= 2 + \cancel{1} + 2 - \cancel{1}$$
$$\boxed{p(1) = 4}$$

$$p(2) = 2 + (2) + 2(2)^2 - (2)^3$$
$$= 2 + 2 + \cancel{2(4)} - \cancel{8}$$
$$= 2 + 2$$
$$\boxed{p(2) = 4}$$

HW

(iii) $p(x) = (x-1)(x+1)$

(iv) $p(x) = x^3$

3) Verify whether the following are zeroes of the polynomial, indicated against them.

(i) $p(x) = 3x+1$, $x = \frac{-1}{3}$.

soln:-

$$Gn, \quad p(x) = 3x+1$$

$$x = \frac{-1}{3}, \quad P\left(\frac{-1}{3}\right) = \cancel{3}\left(\frac{-1}{\cancel{3}}\right)+1$$

$$= -1+1$$

$$P\left(\frac{-1}{3}\right) = 0$$

$\therefore \frac{-1}{3}$ is a zero of $p(x) = 3x+1$.

(ii) $p(x) = 5x - \pi$, $x = \frac{4}{5}$

soln:-

$$Gn, \quad p(x) = 5x - \pi$$

$$x = \frac{4}{5}, \quad p\left(\frac{4}{5}\right) = \cancel{5}\left(\frac{4}{\cancel{5}}\right) - \pi$$

$$= 4 - \pi$$

$\therefore \frac{4}{5}$ is not a zero of given $p(x)$.

(iii) $p(x) = (x+1)(x-2)$, $x = -1, 2$.

soln:

$$p(-1) = (-1+1)(-1-2)$$
$$= (0)(-3)$$
$$p(-1) = 0$$

$$p(2) = (2+1)(2-2)$$
$$= (3)(0)$$
$$p(2) = 0$$

$\therefore -1, 2$ are zeroes of $p(x)$.

(iv) $p(x) = 3x^2 - 1$, $x = \frac{-1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$.

soln:

$$P\left(\frac{-1}{\sqrt{3}}\right) = 3\left(\frac{-1}{\sqrt{3}}\right)^2 - 1$$
$$= \cancel{3}\left(\frac{1}{\cancel{3}}\right) - 1$$
$$= 1 - 1$$
$$P\left(\frac{-1}{\sqrt{3}}\right) = 0$$

$$P\left(\frac{2}{\sqrt{3}}\right) = 3\left(\frac{2}{\sqrt{3}}\right)^2 - 1$$
$$= \cancel{3}\left(\frac{4}{\cancel{3}}\right) - 1$$
$$P\left(\frac{2}{\sqrt{3}}\right) = 3$$

$\therefore \frac{-1}{\sqrt{3}}$ is a zero of $p(x)$, but $\frac{2}{\sqrt{3}}$ is not a zero of $p(x)$

HW

v) $p(x) = x^2 - 1$, $x = 1, -1$

vi) $p(x) = x^2$, $x = 0$

vii) $p(x) = lx + m$, $x = \frac{-m}{l}$

viii) $p(x) = 2x + 1$, $x = \frac{1}{2}$

4) Find the zero of the polynomial :-

i) $p(x) = x + 5$

soln:-

$$p(x) = x + 5$$

$$\Rightarrow x + 5 = 0$$

$$\boxed{x = -5}$$

$\therefore -5$ is a zero of $p(x)$

ii) $p(x) = 3x - 2$

soln:-

$$p(x) = 3x - 2$$

$$\Rightarrow 3x - 2 = 0$$

$$3x = 2$$

$$\boxed{x = \frac{2}{3}}$$

$\therefore \frac{2}{3}$ is a zero of $p(x)$

iii) $p(x) = cx + d$, $c \neq 0$, c & d are real nos

Soln.-

$$p(x) = cx + d$$

$$\Rightarrow cx + d = 0$$

$$cx = -d$$

$$\boxed{x = \frac{-d}{c}}$$

$\therefore$ $\frac{-d}{c}$ is a zero of $p(x)$.

HW

iv) $p(x) = x - 5$

v) $p(x) = 2x + 5$

vi) $p(x) = 3x$

vii) $p(x) = ax$, $a \neq 0$.

DIVISION OF POLYNOMIALS

Division Algorithm :-

Dividend = (Divisor x Quotient) + Remainder

$$p(x) = g(x) \times q(x) + r(x)$$

$$\Rightarrow \boxed{p(x) = g(x) \cdot q(x) + r(x)}$$

1) Divide $p(x) = -x^2 + 2x^3 - 2x - 7$ by $g(x) = -2 + x$.

Soln:

Given $p(x) = -x^2 + 2x^3 - 2x - 7$ (Dividend)

$g(x) = -2 + x$ (Divisor)

$$\begin{array}{r|l} & 2x^2 + 3x + 4 \\ \hline x-2 & 2x^3 - 1x^2 - 2x - 7 \\ & \overset{(-)}{2x^3} \overset{(+)}{-} 4x^2 \\ \hline & \quad 3x^2 - 2x \\ & \quad \overset{(-)}{3x^2} \overset{(+)}{-} 6x \\ \hline & \qquad 4x - 7 \\ & \qquad \overset{(-)}{4x} \overset{(+)}{-} 8 \\ \hline & \qquad\quad 1 \end{array}$$

$\frac{2x^3}{x} = 2x^2$

$\frac{3x^2}{x} = 3x$

$\frac{4x}{x} = 4$

$\therefore q(x) = 2x^2 + 3x + 4$

$r(x) = 1$.

2) Divide $f(x) = 3y^4 - 8y^3 - y^2 - 5y - 5$ by $y - 3$.

Soln:

Gn $f(x) = 3y^4 - 8y^3 - y^2 - 5y - 5$

$g(x) = y - 3$

$$
\begin{array}{r|l}
 & 3y^3 + y^2 + 2y + 1 \\ \hline
y-3 & 3y^4 - 8y^3 - y^2 - 5y - 5 \\
 & \underline{\overset{(-)}{3y^4} \overset{(+)}{-} 9y^3} \\
 & \qquad y^3 - y^2 \\
 & \qquad \underline{\overset{(-)}{y^3} \overset{(+)}{-} 3y^2} \\
 & \qquad\qquad 2y^2 - 5y \\
 & \qquad\qquad \underline{\overset{(-)}{2y^2} \overset{(+)}{-} 6y} \\
 & \qquad\qquad\qquad y - 5 \\
 & \qquad\qquad\qquad \underline{\overset{(-)}{y} \overset{(+)}{-} 3} \\
 & \qquad\qquad\qquad\quad -2
\end{array}
$$

$$\frac{3y^{\cancel{4}\,3}}{\cancel{y}} = 3y^3$$

$$\frac{y^{\cancel{3}\,2}}{\cancel{y}} = y^2$$

$$\frac{2y^{\cancel{2}}}{\cancel{y}} = 2y$$

$$\frac{\cancel{y}}{\cancel{y}} = 1$$

$\therefore \; q(x) = 3y^3 + y^2 + 2y + 1$

$r(x) = -2$.

$\boxed{\text{HW}}$

3) Divide $f(x) = 9x^3 - 5 + x - 3x^2$ by

$g(x) = 3x - 2$.

REMAINDER THEOREM

If $p(x)$ is a polynomial of degree greater than or equal to '1' and is divided by $(x-a)$, then the remainder is $p(a)$.

EXERCISE 2.3 [Pg:40]

1) Find the remainder when x^3+3x^2+3x+1 is divided by

(i) $x+1$,

soln:

$p(x) = x^3+3x^2+3x+1$

$g(x) = x+1$

$\Rightarrow x+1 = 0$

$\boxed{x=-1}$

$\therefore p(-1) = (-1)^3+3(-1)^2+3(-1)+1$

$= \cancel{-1} + \cancel{3} - \cancel{3} + \cancel{1}$

$p(-1) = 0$

$\therefore$ Remainder $= 0$.

ii) $x+\pi$

soln:

$p(x) = x^3+3x^2+3x+1$

$g(x) = x+\pi$.

$\Rightarrow x+\pi = 0$

$\boxed{x=-\pi}$

$\therefore p(-\pi) = (-\pi)^3+3(-\pi)^2+3(-\pi)+1$

Remainder $= -\pi^3+3\pi^2-3\pi+1$

(iii) $5+2x$.

soln.

$$p(x) = x^3+3x^2+3x+1.$$

$$g(x) = 2x+5.$$

$$\Rightarrow 2x+5=0$$

$$2x=-5$$

$$\boxed{x=\frac{-5}{2}}$$

$$\therefore p\left(\frac{-5}{2}\right) = \left(\frac{-5}{2}\right)^3+3\left(\frac{-5}{2}\right)^2+3\left(\frac{-5}{2}\right)+1$$

$$= \frac{-125}{8}+3\left(\frac{25}{4}\right)+\left(\frac{-15}{2}\right)+1$$

$$= \frac{-125}{8}+\frac{75}{4\times2}^{\times2}-\frac{15}{2\times4}^{\times4}+1^{\times8}$$

LCM of 2, 4 & 8 = 8

$$= \frac{-125+150-60+8}{8}$$

$$\therefore \text{Remainder} = \frac{-27}{8}$$

HW

iv) $x-\frac{1}{2}$

v) x.

2) Find the remainder when $x^3 - ax^2 + 6x - a$ is divided by $x - a$.

soln:

$$p(x) = x^3 - ax^2 + 6x - a$$

$$g(x) = x - a$$

$$\Rightarrow x - a = 0$$

$$\boxed{x = a}$$

$$\therefore p(a) = (a)^3 - a(a)^2 + 6(a) - a$$

$$= \cancel{a^3} - \cancel{a^3} + 6a - a$$

$$\boxed{p(a) = 5a}$$

$\therefore$ Remainder $= 5a$

3) Check whether $7 + 3x$ is a factor of $3x^3 + 7x$

soln:

$$p(x) = 3x^3 + 7x$$

$$g(x) = 7 + 3x$$

$$\Rightarrow 7 + 3x = 0$$

$$3x = -7$$

$$\boxed{x = \frac{-7}{3}}$$

$$\therefore p\left(\frac{-7}{3}\right) = 3\left(\frac{-7}{3}\right)^3 + 7\left(\frac{-7}{3}\right)$$

$$= \cancel{3}\left(\frac{-343}{\cancel{27}_9}\right) + \frac{-49}{3}$$

$$= \frac{-343}{9} - \frac{49}{3}.$$

$$= \frac{-343 - 147}{9} \quad (LCM\ of\ 3\ \&\ 9 = 9)$$

$$= \frac{-490}{9} \neq 0.$$

$\therefore$ $7 + 3x$ is not a factor of $P(x)$

HW

1) Find the remainder when

(a) $f(x) = x^3 + 4x^2 - 3x + 10$

$g(x) = x + 4$.

(b) $f(x) = x^4 - 3x^2 + 4$

$g(x) = x - 2$.

FACTOR THEOREM

* $(x-a)$ is a factor of a polynomial $p(x)$, if $p(a) = 0$

* $p(a) = 0$, if $(x-a)$ is a factor of $p(x)$.

EXERCISE 2.4

1) Determine which of the following polynomial has $(x+1)$ a factor:

(i) $x^3 + x^2 + x + 1$

soln:-

$$p(x) = x^3 + x^2 + x + 1$$

$$g(x) = x+1$$

$$\Rightarrow x+1 = 0$$

$$\boxed{x = -1}$$

$$P(-1) = (-1)^3 + (-1)^2 + (-1) + 1$$

$$= \cancel{-1} + \cancel{1} - \cancel{1} + \cancel{1}$$

$$P(-1) = 0$$

Hence $(x+1)$ is a factor of $P(x)$

(ii) $x^4 + x^3 + x^2 + x + 1$

soln:-

$$P(x) = x^4 + x^3 + x^2 + x + 1$$

$$g(x) = x+1.$$

$$\Rightarrow x+1 = 0$$

$$\boxed{x = -1}$$

$$P(-1) = (-1)^4 + (-1)^3 + (-1)^2 + (-1) + 1$$

$$= 1 \cancel{-1} \cancel{+1} \cancel{-1} \cancel{+1}$$

$$\boxed{P(-1) = 1}$$

Hence $(x+1)$ is not a factor of $P(x)$.

(iii) $x^3 - x^2 - (2+\sqrt{2})x + \sqrt{2}$.

Soln:- $p(x) = x^3 - x^2 - (2+\sqrt{2})x + \sqrt{2}$.

$g(x) = x + 1$

$\Rightarrow x + 1 = 0$

$\boxed{x = -1}$

$p(-1) = (-1)^3 - (-1)^2 - (2+\sqrt{2})(-1) + \sqrt{2}$

$= \cancel{-1} \cancel{-1} + \cancel{2} + \sqrt{2} + \sqrt{2}$

$\boxed{p(-1) = 2\sqrt{2}.}$

Hence $(x+1)$ is not a factor of $p(x)$

HW

(iv) $x^4 + 3x^3 + 3x^2 + x + 1$.

2) Use the Factor theorem to determine whether $g(x)$ is a factor of $p(x)$ in each of the following cases:-

(i) $p(x) = 2x^3 + x^2 - 2x - 1$, $g(x) = (x+1)$

soln:- $p(x) = 2x^3 + x^2 - 2x - 1$

$g(x) = x + 1$

$\Rightarrow x + 1 = 0$

$x = -1$

$$p(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1$$

$$= \cancel{-2} + \cancel{1} + \cancel{2} - \cancel{1}$$

$$p(-1) = 0$$

$\therefore$ $(x+1)$ is a factor of $p(x)$.

HW

(ii) $p(x) = x^3 + 3x^2 + 3x + 1$, $g(x) = x + 2$.

(iii) $p(x) = x^3 - 4x^2 + x + 6$, $g(x) = x - 3$

3) Find the value of k, if $(x-1)$ is a factor of $p(x)$ in each of the following:-

(i) $p(x) = x^2 + x + k$

soln:

$$p(x) = x^2 + x + k$$

$$g(x) = x - 1$$

$$\Rightarrow x - 1 = 0$$

$$\boxed{x = 1}$$

$$\therefore p(1) = 0 \Rightarrow (1)^2 + 1 + k = 0$$

$$1 + 1 + k = 0$$

$$2 + k = 0$$

$$\boxed{k = -2}$$

(ii) $p(x) = 2x^2 + kx + \sqrt{2}$

soln:

$p(x) = 2x^2 + kx + \sqrt{2}$

$g(x) = x - 1$

$\Rightarrow x - 1 = 0$

$\boxed{x = 1}$

$\therefore P(1) = 0 \Rightarrow 2(1) + k(1) + \sqrt{2} = 0$

$2 + k + \sqrt{2} = 0$

$k = -2 - \sqrt{2}$

$\boxed{k = -(2 + \sqrt{2})}$

HW

(iii) $p(x) = kx^2 - \sqrt{2}x + 1$

(iv) $p(x) = kx^2 - 3x + k$

4) Factorise

(i) $12x^2 - 7x + 1$

soln:

$12x^2 - 7x + 1 = 12x^2 - 4x - 3x + 1$

$= 4x(3x - 1) - 1(3x - 1)$

Ans $= (4x - 1)(3x - 1)$

(12×1)

Product = 12

-4	-3

Sum = -7

(ii) $2x^2+7x+3$

soln:

$2x^2+7x+3 = 2x^2+x+6x+3$

$= x(2x+1)+3(2x+1)$

$= (2x+1)(x+3)$

$P = 6 \ (2\times3)$

$\frac{1}{2x}$	$\frac{\cancel{6}^{3}}{\cancel{2x}_{1}}$

$S = 7$

(OR)

$2x^2+7x+3 = (2x+1)(x+3)$

$P=6$

1	6

$S=7$

HW

(iii) $6x^2+5x-6$

(iv) $3x^2-x-4$

5) Factorise

(i) x^3-2x^2-x+2

soln:

Let $p(x) = x^3-2x^2-x+2$

Factors of $2 = \{1, -1, 2, -2\}$.

$\therefore \ p(1) = 1^3-2(1)^2-(1)+2$

$= \cancel{1}-\cancel{2}-\cancel{1}+\cancel{2}$

$p(1)=0$

$\Rightarrow x=1$

$\therefore \ x-1$ is a factor of $p(x)$

$$\begin{array}{r|rrrr} 1 & 1 & -2 & -1 & 2 \\ & 0 & 1 & -1 & -2 \\ \hline & 1 & -1 & -2 & \boxed{0} \end{array}$$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ x^2 & x & \text{constant} \end{array}$$

$\therefore\ x^3 - 2x^2 - x + 2 = (x-1)(x^2 - x - 2)$

Now, $x^2 - x - 2 = (x+1)(x-2)$.

$$\begin{array}{c|c} \multicolumn{2}{c}{P=-2} \\ \hline 1 & -2 \\ \hline \multicolumn{2}{c}{S=-1} \end{array}$$

$\therefore\ x^3 - 2x^2 - x + 2 = (x-1)(x+1)(x-2)$

2(ii) $x^3 - 3x^2 - 9x - 5$

soln: Let $p(x) = x^3 - 3x^2 - 9x - 5$

Factors of $5 = \{+1, -1, 5, -5\}$.

$\therefore\ p(1) = (1)^3 - 3(1)^2 - 9(1) - 5$

$= 1 - 3 - 9 - 5$

$\neq 0$.

$p(-1) = (-1)^3 - 3(-1)^2 - 9(-1) - 5$

$= -1 - 3 + 9 - 5$

$p(-1) = 0$

$\Rightarrow x = -1$

$\therefore\ x+1$ is a factor of $p(x)$

$$\begin{array}{r|rrrr} -1 & 1 & -3 & -9 & -5 \\ & 0 & -1 & 4 & 5 \\ \hline & 1 & -4 & -5 & \boxed{0} \end{array}$$

$\therefore\ x^3 - 3x^2 - 9x - 5 = (x+1)(x^2 - 4x - 5)$

Now, $x^2 - 4x - 5 = (x+1)(x-5)$

$$\begin{array}{c|c} \multicolumn{2}{c}{P=-5} \\ \hline +1 & -5 \\ \hline \multicolumn{2}{c}{S=-4} \end{array}$$

$\therefore x^3 - 3x^2 - 9x - 5 = (x+1)(x+1)(x-5)$.

HW

(iii) $x^3 + 13x^2 + 32x + 20$

(iv) $2y^3 + y^2 - 2y - 1$.

ALGEBRAIC IDENTITIES

1) $(a+b)^2 = a^2+2ab+b^2$

2) $(a-b)^2 = a^2-2ab+b^2$

3) $a^2-b^2 = (a+b)(a-b)$

4) $(x+a)(x+b) = x^2+(a+b)x+ab$.

5) $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

6) $(a+b)^3 = a^3+b^3+3a^2b+3ab^2$ (or)
$a^3+b^3+3ab(a+b)$

7) $(a-b)^3 = a^3-b^3-3a^2b+3ab^2$ (or)
$a^3-b^3-3ab(a-b)$

8) $a^3+b^3 = (a+b)(a^2-ab+b^2)$

9) $a^3-b^3 = (a-b)(a^2+ab+b^2)$

10) $a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2 - ab-bc-ac)$

11) $a^3+b^3+c^3 = 3abc$ [if $a+b+c=0$]

EXERCISE 2.5 [Pg: 48]

1) Use suitable identities to find the following products.

(i) $(x+4)(x+10)$

soln:

WKT $(x+a)(x+b) = x^2+(a+b)x+ab$.

Here $a=4$, $b=10$.

$\therefore (x+4)(x+10) = x^2+(4+10)x+(4)(10)$

Ans $= x^2+14x+40$.

ii) $(3x+4)(3x-5)$

soln:

WKT $(x+a)(x+b) = x^2+(a+b)x+ab$.

Here $a=4$; $b=-5$; $x=(3x)$

$\therefore (3x+4)(3x-5) = (3x)^2+(4-5)(3x)+(4)(-5)$

$= 9x^2+(-1)(3x)-20$

Ans $= 9x^2-3x-20$

iii) $\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)$.

soln:

WKT $(a+b)(a-b) = a^2-b^2$.

Here $a=y^2$, $b=\frac{3}{2}$.

$\therefore \left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right) = (y^2)^2-\left(\frac{3}{2}\right)^2$

Ans $= y^4-\frac{9}{4}$.

HW

iv) $(x+8)(x-10)$

v) $(3-2x)(3+2x)$

2) Evaluate the following products without multiplying directly.

(i) 103×107

soln:

METHOD: 1

$103 \times 107 = (100+3)(100+7)$

WKT $(x+a)(x+b) = x^2 + (a+b)x + ab$

$\therefore (100+3)(100+7) = (100)^2 + [3+7](100) + (3)(7)$ $\left[\begin{array}{l} x = 100 \\ a = 3 \\ b = 7 \end{array}\right.$

$= 10000 + 1000 + 21$

Ans $= 11021$

METHOD: 2

$103 \times 107 = (105-2)(105+2)$

WKT $(a+b)(a-b) = a^2 - b^2$ $\left[\begin{array}{l} a = 105 \\ b = 2 \end{array}\right.$

$\therefore (105+2)(105-2) = (105)^2 - (2)^2$

$= 11025 - 4$

Ans $= 11021$

$$\begin{array}{r} 105 \\ 105 \\ \hline 525 \\ 000 \\ 105 \\ \hline 11025 \\ -4 \\ \hline 11021 \end{array}$$

(ii) 95×96

soln:

$95 \times 96 = (100-5)(100-4)$

WKT $(x+a)(x+b) = x^2 + (a+b)x + ab$

$\therefore (100-5)(100-4) = (100)^2 + [-5-4](100) + (-5)(-4)$ $\left[\begin{array}{l} x = 100 \\ a = -5 \\ b = -4 \end{array}\right.$

$= 10000 - 900 + 20$

Ans $= 9120$

$\boxed{\text{HW}}$

iii) 104×96.

3) Factorize the following using appropriate identities

i) $9x^2 + 6xy + y^2$

soln:
$= 9x^2 + 6xy + y^2$

$= (3x)^2 + 2(3x)(y) + y^2$ $[\because a^2 + 2ab + b^2 = (a+b)^2$

Ans $= (3x+y)^2$

ii) $x^2 - \frac{y^2}{100}$.

soln:

$x^2 - \frac{y^2}{100} = x^2 - \left(\frac{y}{10}\right)^2$

Ans $= \left(x + \frac{y}{10}\right)\left(x - \frac{y}{10}\right)$

$[\because (a^2 - b^2 = (a+b)(a-b)$

$\boxed{\text{HW}}$

iii) $4y^2 - 4y + 1$.

4) Expand each of the following using suitable identities

(i) $(x+2y+4z)^2$.

soln:-

WKT $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

$\therefore (x+2y+4z)^2 = x^2+(2y)^2+(4z)^2+2(x)(2y)+2(2y)(4z)+2(4z)(x)$ $\left[\begin{array}{l} a=x \\ b=2y \\ c=4z \end{array}\right.$

Ans $= x^2+4y^2+16z^2+4xy+16yz+8xz$.

(ii) $(-2x+3y+2z)^2$

soln:-

WKT $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

$\left[\begin{array}{l} a=-2x \\ b=3y \\ c=2z \end{array}\right.$

$\therefore (-2x+3y+2z)^2 = (-2x)^2+(3y)^2+(2z)^2+2(-2x)(3y)+2(3y)(2z)+2(2z)(-2x)$

Ans $= 4x^2+9y^2+4z^2-12xy+12yz-8xz$

(iii) $\left[\frac{1}{4}a-\frac{1}{2}b+1\right]^2$

soln:-

WKT $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

$\left[\begin{array}{l} a=\frac{1}{4}a \\ b=\frac{-1}{2}b \\ c=1 \end{array}\right.$

$\therefore \left[\frac{1}{4}a - \frac{1}{2}b + 1\right]^2 = \left(\frac{1}{4}a\right)^2 + \left(\frac{-1}{2}b\right)^2 + (1)^2 + \cancel{2}\left(\frac{1}{4}a\right)\left(\frac{-1}{\cancel{2}}b\right) +$

$$\cancel{2}\left(\frac{-1}{\cancel{2}}b\right)(1) + \cancel{2}(1)\left(\frac{1}{\cancel{4}_2}a\right)$$

$$= \frac{1}{16}a^2 + \frac{1}{4}b^2 + 1 - \frac{ab}{4} - b + \frac{a}{2}.$$

$$\text{Ans} = \frac{a^2}{16} + \frac{b^2}{4} + 1 - \frac{ab}{4} - b + \frac{a}{2}.$$

$\boxed{\text{HW}}$

iv) $(2x - y + z)^2$

v) $(3a - 7b - c)^2$.

vi) $(-2x + 5y - 3z)^2$.

5) Factorise

(i) $4x^2+9y^2+16z^2+12xy-24yz-16xz$.

soln:-

$$= (2x)^2+(3y)^2+(-4z)^2+2(2x)(3y)+2(3y)(-4z)$$
$$+2(2x)(-4z)$$

Ans $= (2x+3y-4z)^2$

ii) $2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$

soln:

$$= (-\sqrt{2}x)^2+y^2+(2\sqrt{2}z)^2+2(-\sqrt{2}x)(y)+$$
$$2(y)(2\sqrt{2}z)+2(-\sqrt{2}x)(y)$$

$Ans = (-\sqrt{2}x + y + 2\sqrt{2}z)^2$

$\left[\begin{array}{l} -\sqrt{2}x - \sqrt{2} = (\sqrt{2})^2 = 2 \\ \sqrt{8} = \sqrt{4\times 2} \\ = \underline{\sqrt{2}\times\sqrt{2}}\times\sqrt{2} = 2\sqrt{2} \end{array}\right.$

6) Write the following cubes in expanded form.

(i) $(2x+1)^3$

soln:

WKT $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

Here $a = 2x$; $b = 1$

$\therefore (2x+1)^3 = (2x)^3 + (1)^3 + 3(2x)(1)\,[2x+1]$

$= 8x^3 + 1 + 6x(2x+1)$

$= 8x^3 + 1 + 12x^2 + 6x$

$Ans = 8x^3 + 12x^2 + 6x + 1$

HW

(ii) $(2a - 3b)^3$ (iv) $\left[x - \frac{2}{3}y\right]^3$

soln:

WKT

(iii) $\left[\frac{3}{2}x + 1\right]^3$

soln:

WKT $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

Here $a = \frac{3}{2}x$; $b = 1$

$\left(\frac{3}{2}x + 1\right)^3 = \left(\frac{3}{2}x\right)^3 + (1)^3 + 3\left(\frac{3}{2}x\right)(1)\left[\frac{3}{2}x + 1\right]$

$= \frac{27}{8}x^3 + 1 + \frac{9}{2}x\left[\frac{3}{2}x + 1\right]$

$= \frac{27}{8}x^3 + 1 + \frac{27}{4}x^2 + \frac{9}{2}x$

Ans. $= \frac{27}{8}x^3 + \frac{27}{4}x^2 + \frac{9}{2}x + 1$

7) Evaluate

(i) $(99)^3$

soln:

$99 = 100 - 1$

$\therefore 99^3 = (100-1)^3$

WKT $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

Here $a = 100$; $b = 1$

$(100-1)^3 = (100)^3 - (1)^3 - 3(100)(1)(100-1)$

$= 1000000 - 1 - 300(99)$ $\quad 99 \times 3 = 297$

$= 1000000 - 1 - 29700$

$= 1000000 - 29701$

Ans $= 970299$

(ii) $(102)^3$

soln:

$102 = 100 + 2$

$\therefore (102)^3 = (100+2)^3$

WKT $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

Here $a = 100$; $b = 2$

$\therefore (100+2)^3 = (100)^3 + (2)^3 + 3(100)(2)[100+2]$

$= 1000000 + 8 + 600(102)$

$= 1000000 + 8 + 61200$

Ans $= 10,61,208$

$\boxed{HW}$

iii) $(998)^3$

8) <u>Factorise the following</u>

i) $8a^3 + b^3 + 12a^2b + 6ab^2$

<u>soln</u>:

$(2a)^3 + (b)^3 + 3(2a)^2(b) + 3(2a)(b)^2$ $\quad \left[\begin{array}{l} a = 2a \\ b = b. \end{array} \right.$

$= (2a+b)^3$; (or)

Ans $= (2a+b)(2a+b)(2a+b)$

ii) $27 - 125a^3 - 135a + 225a^2.$

<u>soln</u>:

$= (3)^3 + (-5a)^3 + 3(3)^2(-5a) + 3(3)(-5a)^2$

Ans $= [3-5a]^3$ or $\quad \left[\begin{array}{l} a = 3 \\ b = -5a. \end{array} \right.$

$= (3-5a)(3-5a)(3-5a)$

$\boxed{HW}$

iii) $8a^3 - b^3 - 12a^2b + 6ab^2$

iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2.$

v) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

Soln:-

$$= (3p)^3 + \left(\frac{-1}{6}\right)^3 + 3(3p)^2\left(-\frac{1}{6}\right) + 3(3p)\left(\frac{1}{6}\right)^2$$

$$\left[\begin{array}{l} a = 3p \\ b = \frac{-1}{6} \end{array}\right.$$

Ans $= \left(3p - \frac{1}{6}\right)^3$

9) Verify

i) $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$

Verification:-

$$RHS = (x+y)(x^2 - xy + y^2)$$

$$= x^3 - \cancel{x^2y} + \cancel{xy^2} + \cancel{yx^2} - \cancel{xy^2} + y^3$$

$$= x^3 + y^3$$

RHS = LHS.

Hence verified.

$\boxed{HW}$

ii) $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

10) Factorise:- [Hint Q:9]

(i) $27y^3 + 125z^3$.

soln:

$$27y^3 + 125z^3 = (3y)^3 + (5z)^3$$

WKT $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$

Here $x = 3y$; $y = 5z$.

$$\therefore (3y)^3 + (5z)^3 = (3y+5z)\left[(3y)^2 - 3y(5z) + (5z)^2\right]$$

$$\text{Ans} = (3y+5z)\left[9y^2 - 15yz + 25z^2\right]$$

(ii) $64m^3 - 343n^3$

soln:

$$64m^3 - 343n^3 = (4m)^3 - (7n)^3$$

WKT $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$$\therefore (4m)^3 - (7n)^3 = (4m - 7n)\left[(4m)^2 + (4m)(7n) + (7n)^2\right]$$

$$\text{Ans} = (4m - 7n)\left[16m^2 + 28mn + 49n^2\right]$$