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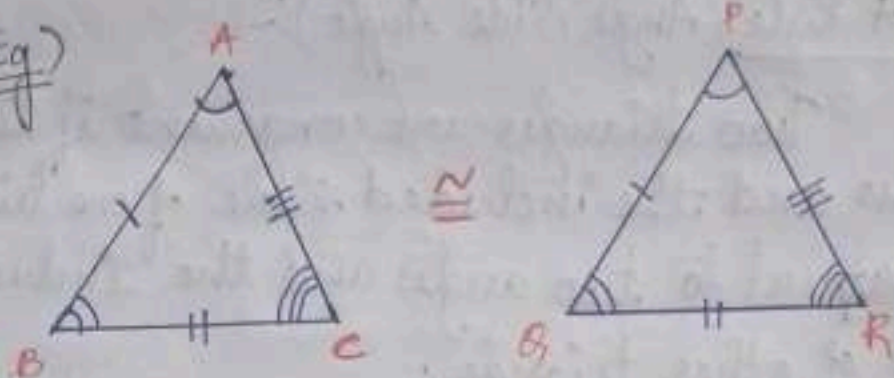
Chapter - 7  
Triangles

→ Notes :-

- \* Two geometric figures are congruent if they have the same shape and same size.
- \* Two triangles are congruent if three sides of one triangle are equal to the three sides of other triangle.
- \* The symbol used for congruence is  $\cong$ .
- \* Two line segments are congruent if they have the same length.
- \* Two angles are congruent if they have equal measures.
- \* Two circles are congruent if they have equal radii.
- \* Two squares are congruent if they have equal sides.
- \* Two triangles are congruent if and only if 3 sides and angles of one triangle are congruent

to the corresponding sides and angles of another triangle.

(Ex)



In  $\triangle ABC$  and  $\triangle PQR$ ,

$$A \leftrightarrow P$$

$$B \leftrightarrow Q$$

$$C \leftrightarrow R$$

$$\overline{AB} \leftrightarrow \overline{PQ}$$

$$\overline{BC} \leftrightarrow \overline{QR}$$

$$\overline{AC} \leftrightarrow \overline{PR}$$

$$\times \left\{ \begin{array}{l} \triangle ABC \cong \triangle PQR \\ \triangle BCA \cong \triangle PRQ \end{array} \right.$$

Hence  $\triangle ABC \cong \triangle PQR$ .

\* The corresponding parts of congruent triangles (CPCT) are equal.

⇒ Congruent Criteria :-

1) SAS Rule (Side Angle Side) :-

Two triangles are congruent if two sides and the included angle of one triangle

are equal to the two sides and the included angle of the other triangle.

2) ASA Rule (Angle Side Angle):-

Two triangles are congruent if two angles and the included side of one triangle are equal to two angles and the included side of other triangle.

3) AAS Rule (Angle Angle Side):-

Two triangles are congruent if any two pairs of angles and one pair of corresponding sides are equal.

4) SSS Rule (Side Side Side):-

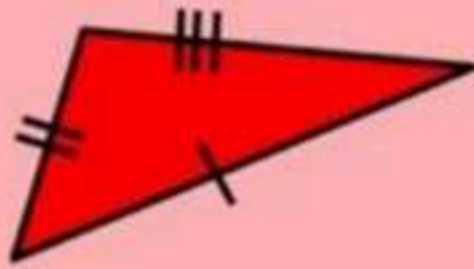
If three sides of one triangle are equal to the three sides of another triangle, then the two triangles are congruent.

5) RHS Rule (Rightangle Hypotenuse Side):-

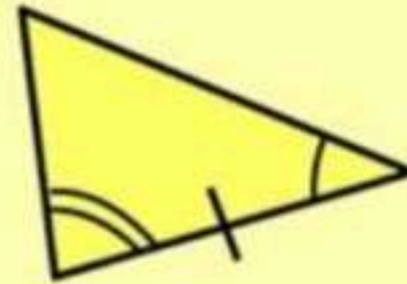
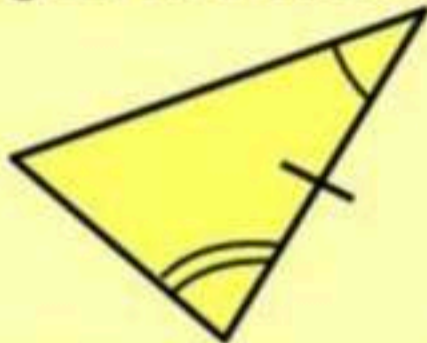
If in two right triangles, the hypotenuse and one side of one triangle are equal to the hypotenuse and one side of the other triangle, then the two triangles are congruent.

Two triangles are congruent if one of the following conditions is satisfied

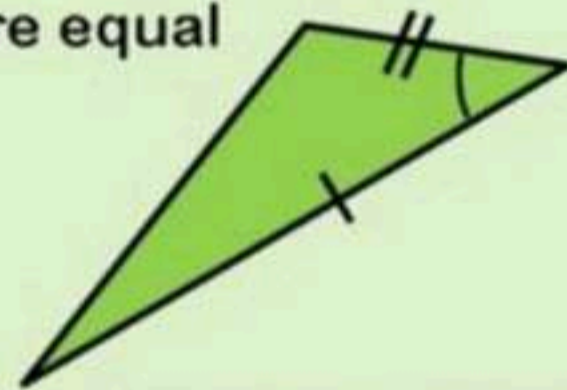
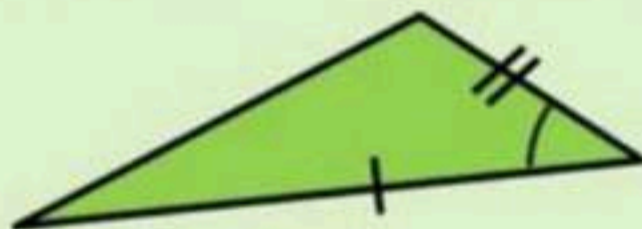
**SSS:** Three sides are equal



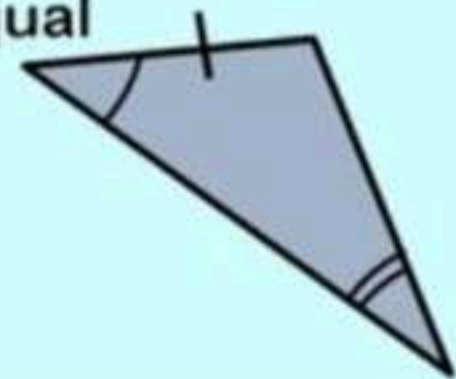
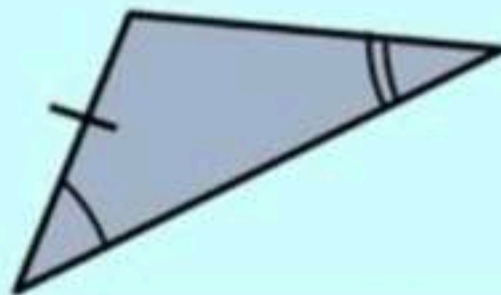
**ASA:** Two angles and the included side are equal



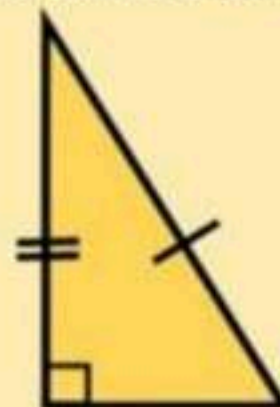
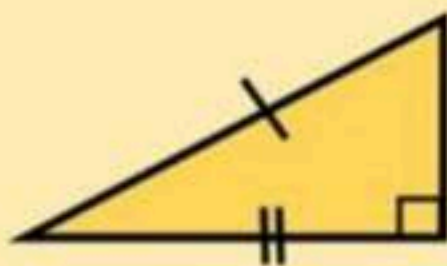
**SAS:** Two sides and the included angle are equal



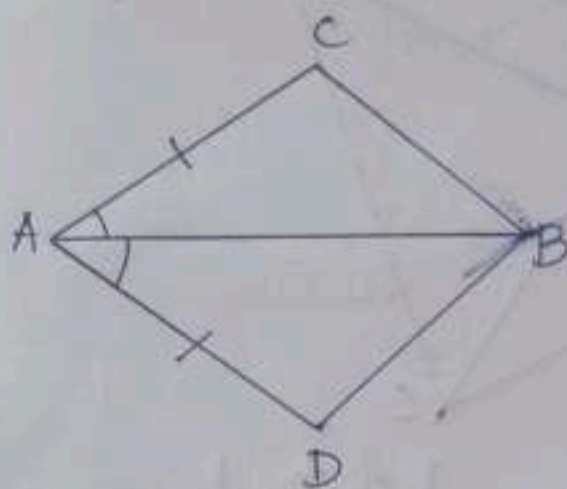
**AAS:** Two angles and an opposite side are equal



**RHS:** A right angle, the hypotenuse and another side are equal



Exercise 7.1 [Pg 118-120]



Given :-

$$AC = AD$$

$AB$  bisects  $\angle A$ .

To prove :-

$$\triangle ABC \cong \triangle ABD.$$

Solution :-

In  $\triangle ABC$  and  $\triangle ABD$ ,

$$AC = AD \text{ (Given)}$$

$\therefore AB$  bisects  $\angle A$ ,

$$\angle CAB = \angle BAD.$$

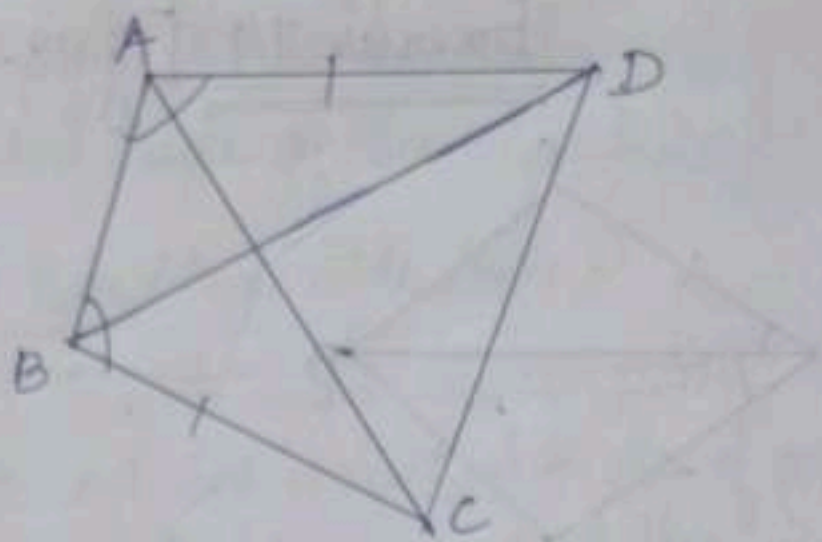
$$AB = BA \text{ (Common Side)}$$

$$\therefore \triangle ABC \cong \triangle ABD \text{ (SAS Rule)}$$

Hence Proved.

Ans :-

$$\Rightarrow BC = BA \text{ (By CPCT)}$$



Given:- ABCD - quadrilateral

$AD = BC$

$\angle DAB = \angle CBA$

To prove:- i)  $\triangle ABD \cong \triangle BAC$

ii)  $BD = AC$

iii)  $\angle ABD = \angle BAC$

Solution:-

i) In  $\triangle ABD$  and  $\triangle BAC$ ,

$AD = BC$  (Given)

$\angle DAB = \angle CBA$  (Given)

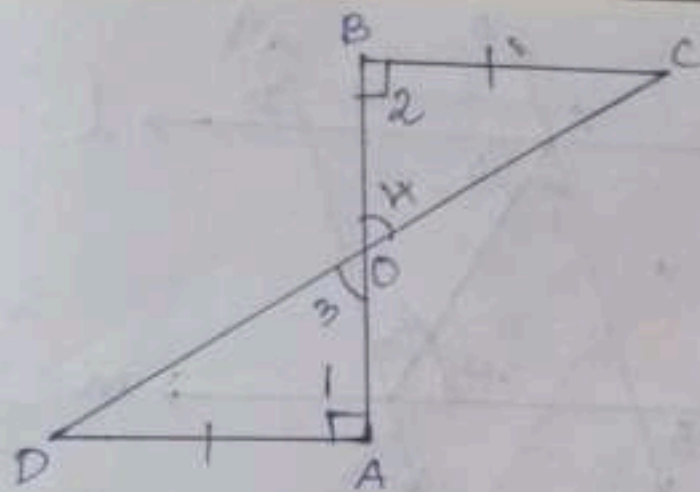
$AB = BA$  (Common)

$\Rightarrow \triangle ABD \cong \triangle BAC$  (By SAS rule)

$\Rightarrow$  ii)  $BD = AC$  (By CPCT)

$\Rightarrow$  iii)  $\angle ABD = \angle BAC$  (By CPCT)

Ans:- Hence Proved



Given:-

$AD \perp AB$  and  $BC \perp AB$ .

To prove:-

CD bisects AB.

Solution:-

$$AD \perp AB \Rightarrow \angle DAB = 90^\circ \Rightarrow \angle OAD = 90^\circ$$

$$BC \perp AB \Rightarrow \angle CBA = 90^\circ \Rightarrow \angle OBC = 90^\circ$$

$$\therefore \angle 1 = \angle 2 = 90^\circ \quad \text{--- (1)}$$

In  $\triangle OBC$ , and  $\triangle OAD$ ,

$$\angle 1 = \angle 2 \quad (\text{from (1)})$$

$$\angle 3 = \angle 4 \quad (\text{Vertically opposite angles})$$

$$BC = DA \quad (\text{Given})$$

$$\therefore \triangle OBC \cong \triangle OAD \quad (\text{By AAS Rule})$$

$$CO = DO \quad \left. \vphantom{CO = DO} \right\} (\text{By CPCT})$$

$$BO = AO.$$

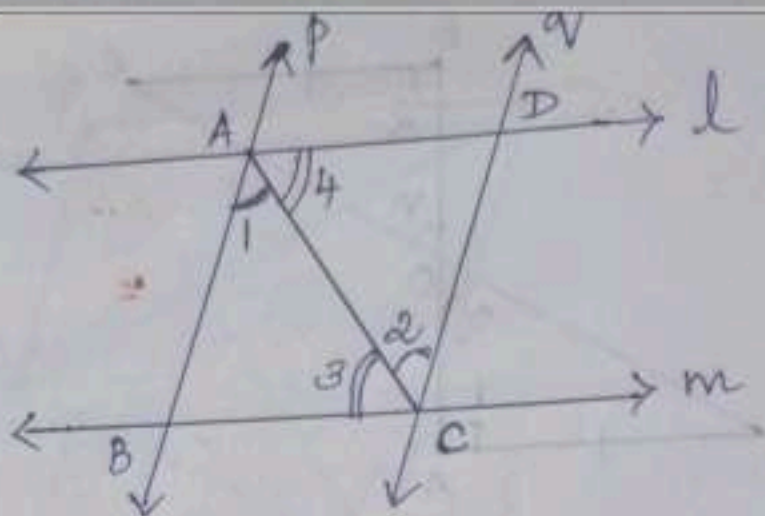
$\therefore$  O is the midpoint of BA and CD.

Thus, CD bisects AB

Hence Proved.

Ans:-

A2



Given :-  
 $l$  and  $m$  are 2 parallel lines.  
 $p$  and  $q$  are 2 parallel lines.

To prove :-  
 $\triangle ABC \cong \triangle CDA$ .

Solution :-

Since  $l$  and  $m$  are 2 parallel lines,  
 and let  $AC$  is a transversal.

$\therefore \angle 1 = \angle 2$  - (1) (Alternate Interior  
 and  $\angle 3 = \angle 4$  - (2) Angles)

In  $\triangle ABC$  and  $\triangle CDA$ ,

$$\angle 1 = \angle 2 \text{ (from (1))}$$

$$\angle 3 = \angle 4 \text{ (from (2))}$$

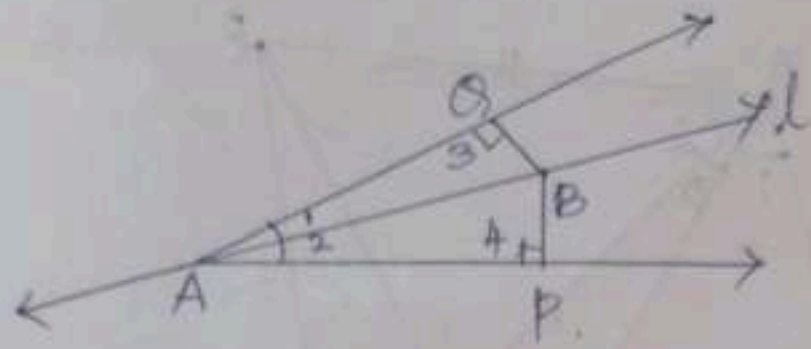
$$AC = CA \text{ (Common)}$$

$\therefore \triangle ABC \cong \triangle CDA$  (By ASA Rule)

Ans :-

Hence Proved.





Given :-

\*  $l$  is the bisector of  $\angle A$ .

\*  $BP$  and  $BQ$  are  $\perp$  from  $B$  to  $\angle A$ .

To prove :-

i)  $\triangle APB \cong \triangle ASB$

ii)  $BP = BQ$  (or)  $B$  is equidistant from the arms of  $\angle A$ .

Solution :-

i) In  $\triangle APB$  and  $\triangle ASB$ ,

$$\angle 1 = \angle 2 \quad (\because l \text{ is the bisector of } \angle A)$$

$$\Rightarrow \angle SAB = \angle BAP$$

$$\angle 3 = \angle 4 \quad (\because \angle ASB = \angle APB = 90^\circ)$$

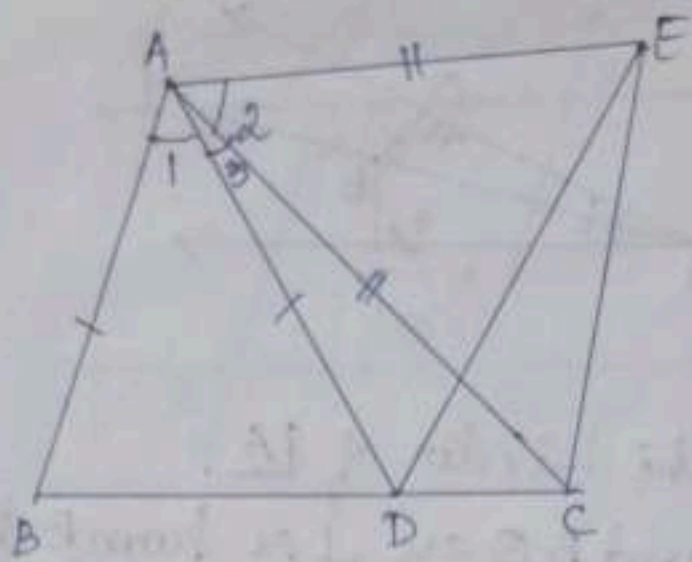
$$AB = AB \quad (\text{Common})$$

$\Rightarrow \therefore \triangle APB \cong \triangle ASB$  (AAS Rule).

ii)  $BP = BQ$  (By CPCT)

$\Rightarrow$  i.e.,  $B$  is equidistant from the arms of  $\angle A$

Ans :- Hence Proved.



Given :-

$AC = AE$

$AB = AD$

$\angle BAD = \angle EAC$

To prove :-

$BC = DE$

Solution :-

Let  $\angle BAC = \angle 1$ ,  $\angle EAC = \angle 2$  and  $\angle CAD = \angle 3$

$\therefore \angle 1 = \angle 2$  (Given)

Adding  $\angle 3$  on both sides,

$\angle 1 + \angle 3 = \angle 2 + \angle 3$

$\angle CAB = \angle EAD$  — (1)

In  $\Delta ABC$  and  $\Delta ADE$ ,

$AB = AD$  (Given)

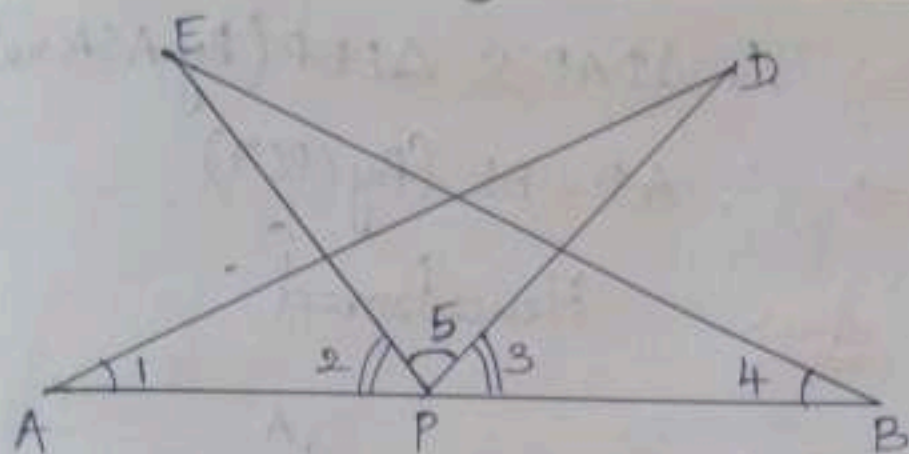
$AC = AE$  (Given)

$\angle BAC = \angle DAE$  (from (1))

$\Rightarrow \therefore \Delta ABC \cong \Delta ADE$  (By SAS Rule)

$\Rightarrow \therefore BC = DE$  (By CPCT)

Ans: Hence Proved



Given :-

P is the midpoint.

$$\angle BAD = \angle ABE$$

$$\angle EPA = \angle DPB$$

To prove :-

i)  $\triangle DAP \cong \triangle EBP$ .

ii)  $AD = BE$

Solution :-

Let  $\angle BAD = \angle 1$ ,  $\angle ABE = \angle 4$ ,  $\angle EPA = \angle 2$

$\angle EPD = \angle 5$  and  $\angle DPB = \angle 3$ .

$\therefore$  P is the midpoint,  $AP = PB$ . (Given) ①

Also,  $\angle 1 = \angle 4$  and  $\angle 2 = \angle 3$ . (Given)

$$\angle 2 = \angle 3$$

Adding  $\angle 5$  on both sides,

$$\angle 2 + \angle 5 = \angle 3 + \angle 5,$$

$$\angle DPA = \angle EPB \quad \text{--- (2)}$$

In  $\triangle DAP$  and  $\triangle EBP$ ,

$AP = BP$  (from ①)

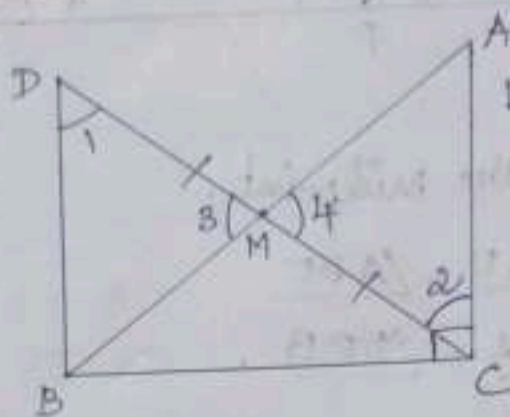
$\angle DPA = \angle EPB$ . (from ②)

$\angle 1 = \angle 4$  (Given)

$\Rightarrow \therefore \triangle DAP \cong \triangle EBP$  (By ASA rule)

$\Rightarrow \therefore AD = BE$  (By CPCT)

Q.E.D. Hence Proved



Let  $\angle DMB = \angle 3$

$\angle AMC = \angle 4$

$\angle BDM = \angle 1$

$\angle ACM = \angle 2$

Given:-

ABC is right triangle right angled at C

$$\Rightarrow \angle ACB = 90^\circ \text{ --- (1)}$$

M is the midpoint of hypotenuse AB

$$\Rightarrow AM = BM \text{ --- (2)}$$

$$DM = CM \text{ --- (3)}$$

To prove:-

i)  $\triangle AMC \cong \triangle BMD$

ii)  $\triangle DBC$  is right angle

iii)  $\triangle DBC \cong \triangle ACB$

$$N) CM = \frac{1}{2} AB.$$

Solution:-

i) In  $\triangle AMC$  and  $\triangle BMD$ ,

$$CM = DM \text{ (from (3))}$$

$$AM = BM \text{ (from (2))}$$

(55)

$$\angle 3 = \angle 4 \quad (\text{Vertically opposite angles})$$

$$\Rightarrow \therefore \triangle AMC \cong \triangle BMD \quad (\text{By SAS rule}).$$

Hence Proved.

From this,

$$\left. \begin{array}{l} AC = BD \quad \text{--- (a)} \\ \angle 1 = \angle 2 \quad \therefore \end{array} \right\} \quad (\text{By CPCT}).$$

Considering AC and BD as equal lines,  
 $\angle 1$  and  $\angle 2$  as alternate interior angles and  
CD as transversal,

$$BD \parallel AC \quad \text{--- (4)}$$

$$\text{ii) } \angle ACB + \angle DBC = 180^\circ \quad (\text{Co interior Angles from (4)}).$$

$$90^\circ + \angle DBC = 180^\circ \quad (\text{from (1)})$$

$$\angle DBC = 180^\circ - 90^\circ$$

$$\angle DBC = 90^\circ$$

$$\Rightarrow \therefore \angle DBC \text{ is a right angle}$$

Hence Proved

$$\text{iii) } \text{In } \triangle DBC \text{ and } \triangle ACB,$$

$$BC = CB \quad (\text{Common})$$

$$AC = BD \quad (\text{from (a)})$$

$$\angle ACB = \angle DBC \quad (= 90^\circ)$$

$$\Rightarrow \therefore \triangle DBC \cong \triangle ACB$$

Hence Proved.

$$AB = CD \text{ (By CPCT)}$$

(5)

⊥ (5)

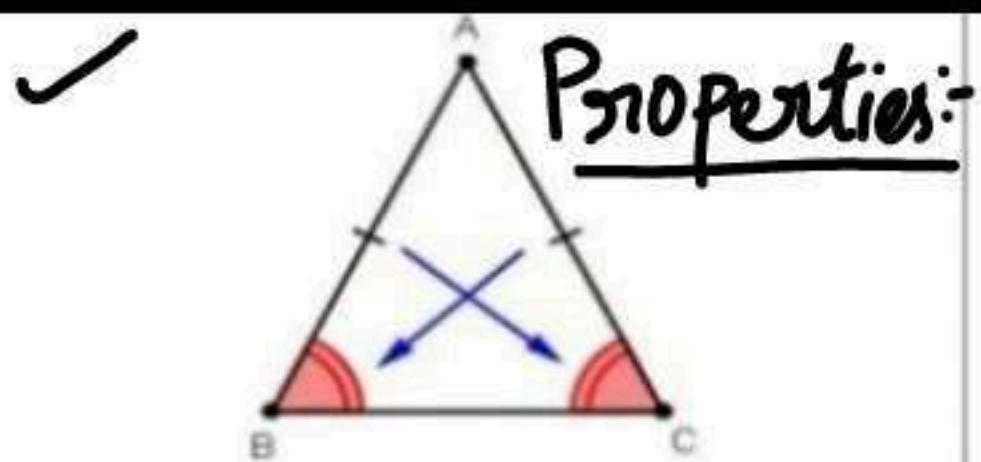
∴ Multiply (5) by  $\frac{1}{2}$ ,

$$(5) \Rightarrow \frac{1}{2} \times AB = \frac{1}{2} \times CD.$$

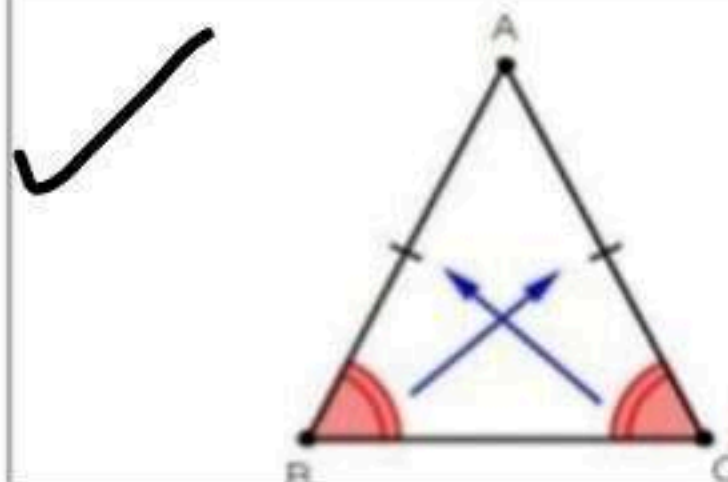
$$CM = \frac{1}{2} AB. \quad [\because M \text{ is the midpoint of } CD \text{ also}]$$

⇒ Hence Proved.

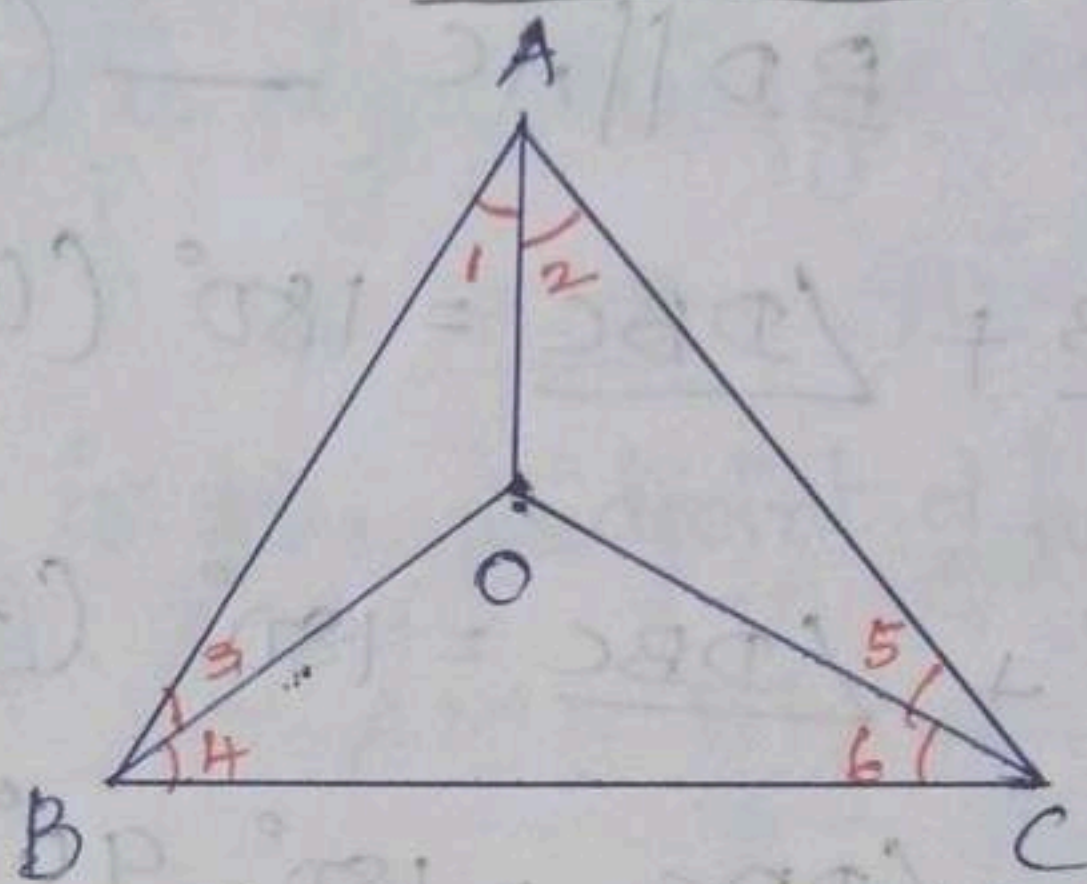
If two sides of a triangle are equal in measure, then the angles opposite those sides are equal in measure



If two angles of a triangle are equal in measure, then the sides opposite those angles are equal in measure



## Exercise 7.2 (Pg 123-124)



Given:-

Isosceles  $\triangle ABC \rightarrow AB = AC$

Bisectors of  $\angle B$  and  $\angle C$  intersect at O.

$\Rightarrow$  In  $\triangle ABC$ ,  $\angle 3 = \angle 4$

In  $\triangle ACB$ ,  $\angle 5 = \angle 6$ .

To prove:-

i)  $OB = OC$

ii) AO bisects  $\angle A$ .

Solution:-

i) In  $\triangle ABC$ ,

$$AB = AC \text{ (Given)}$$

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$$\Rightarrow \angle ACB = \angle ABC \text{ (}\because \text{Angles opposite to equal sides are equal)}$$

$$\frac{1}{2} \angle ACB = \frac{1}{2} \angle ABC \text{ (}\because \text{OB and OC are bisectors)}$$

$$\angle OBC = \angle OCB$$

$$\text{ie, } \angle 4 = \angle 6 \text{ [}\because \angle OBC = \angle 4 \text{ and } \angle OCB = \angle 6 \text{]}$$

$$\Rightarrow OC = OB \text{ [}\because \text{Sides opposite to equal}$$

$$\Rightarrow \text{Hence Proved} \text{ [}\because \text{angles are equal]}$$

ii) In  $\triangle AOB$  and  $\triangle AOC$ ,

$$OB = OC \text{ (from (2))}$$

$$AB = AC \text{ (Given)}$$

$$\angle 3 = \angle 5 \text{ [}\because \angle 3 = \angle 4 \text{ and } \angle 5 = \angle 6 \text{ (Given)}]$$

$$\angle 5 = \angle 6$$

$$\text{also } \angle 4 = \angle 6 \text{ (from (1))}$$

$$\therefore \triangle AOB \cong \triangle AOC \text{ (By SAS Rule)}$$

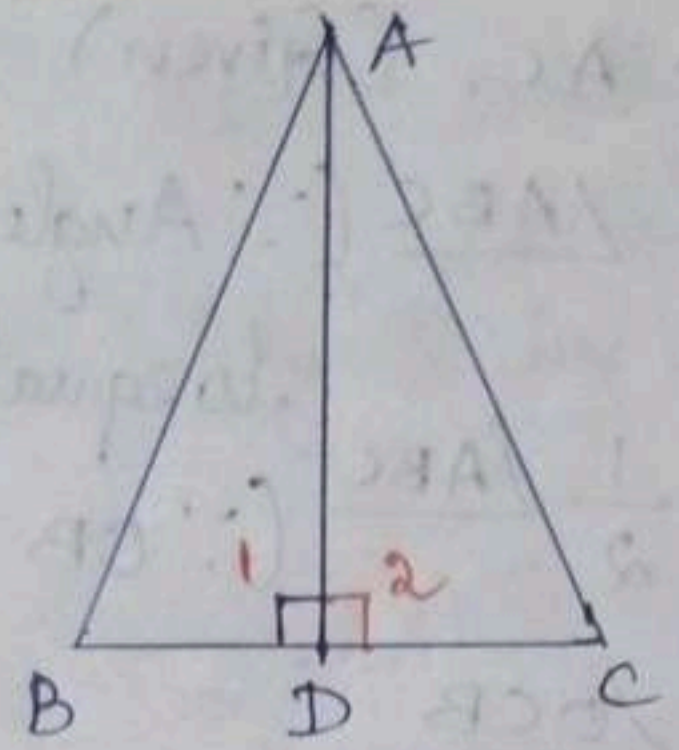
$$\Rightarrow \angle 1 = \angle 2 \text{ (By CPCT)}$$

ie, AO bisects  $\angle A$

$$\Rightarrow \text{Hence Proved}$$



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Given:-

$\therefore AD$  is the perpendicular bisector of  $BC$

$\Rightarrow \angle 1 = \angle 2 = 90^\circ$  — (1)

$\Rightarrow BD = DC$  — (2)

To prove:-

$\Delta ABC$  is isosceles  $\Delta \rightarrow AB = AC$

Solution:-

In  $\Delta ABD$  and  $\Delta ACD$ ,

$AD = AD$  (Common)

$BD = DC$  (from (2))

$\angle 1 = \angle 2$  (from (1))

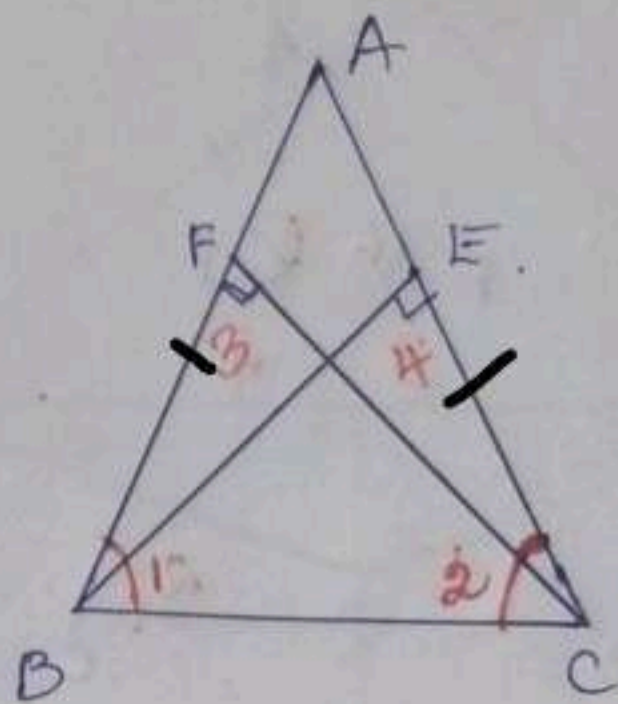
$\therefore \Delta ABD \cong \Delta ACD$  (By SAS rule)

$\Rightarrow AB = AC$  (By CPCT)

Ans:-

ie,  $\Delta ABC$  is an isosceles triangle

Hence Proved.



Given:-

$\triangle ABC$  is an isosceles triangle

$$\Rightarrow AB = AC \text{ --- (1)}$$

CF and BE are altitudes drawn at AB and AC respectively

$$\Rightarrow \angle BFC = \angle CEB = 90^\circ.$$

$$\Rightarrow \angle 3 = \angle 4 = 90^\circ.$$

To prove:-

Altitudes are equal

$$\Rightarrow CF = BE.$$

Solution:-

In  $\triangle ABC$ ,

$$\angle 1 = \angle 2.$$

[from (1) and angles opposite sides are equal]

$\angle 3 = \angle 4$  (2)

In  $\triangle BFC$  and  $\triangle CEB$ ,

$$BC = CB \text{ (Common)}$$

$$\angle 1 = \angle 2 \text{ (from (2))}$$

$$\angle 3 = \angle 4 = 90^\circ$$

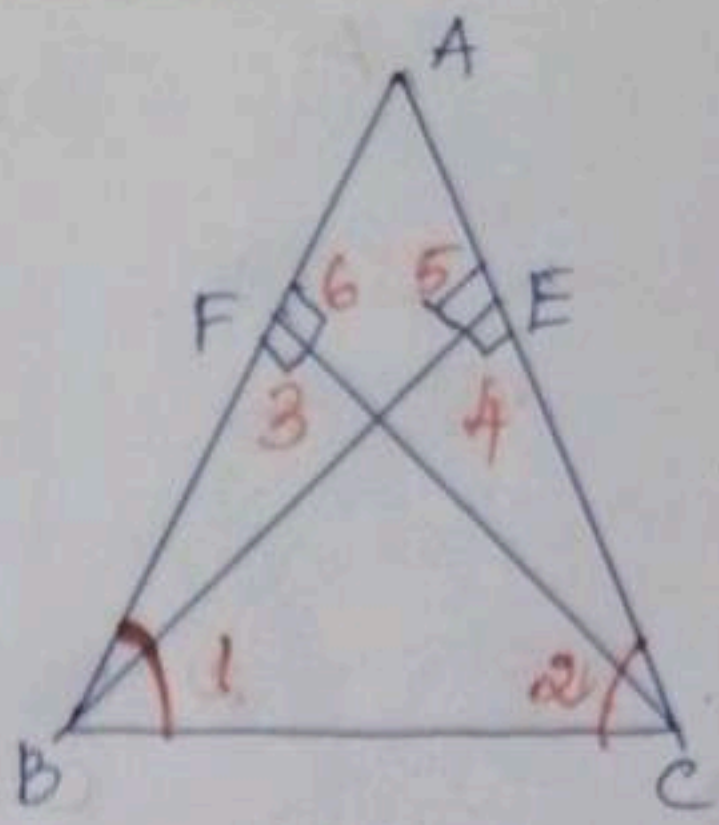
$$BF = CE \text{ (} \because AB = AC \text{)}$$

$\therefore \triangle BFC \cong \triangle CEB$  (By AAS Rule)

$$\Rightarrow \therefore CF = BE \text{ (By CPCT)}$$

Ans:-

Hence Proved



Given :-

CF and BE are altitudes to AB and AC respectively and  $CF = BE$

$\Rightarrow \angle B = \angle C = 90^\circ$  — (1) (Since angles opposite to equal sides are equal)

To prove :-

- i)  $\triangle ABE \cong \triangle ACF$
- ii)  $AB = AC$  or, ABC is an isosceles triangle.

Solution :-

from (1),  
 $\angle B = \angle C = 90^\circ$  — (2)

In  $\triangle ABE$  and  $\triangle ACF$ ,

$BE = CF$  (Given)

$\angle B = \angle C$  (from (2))

$\angle A = \angle A$  (Common)

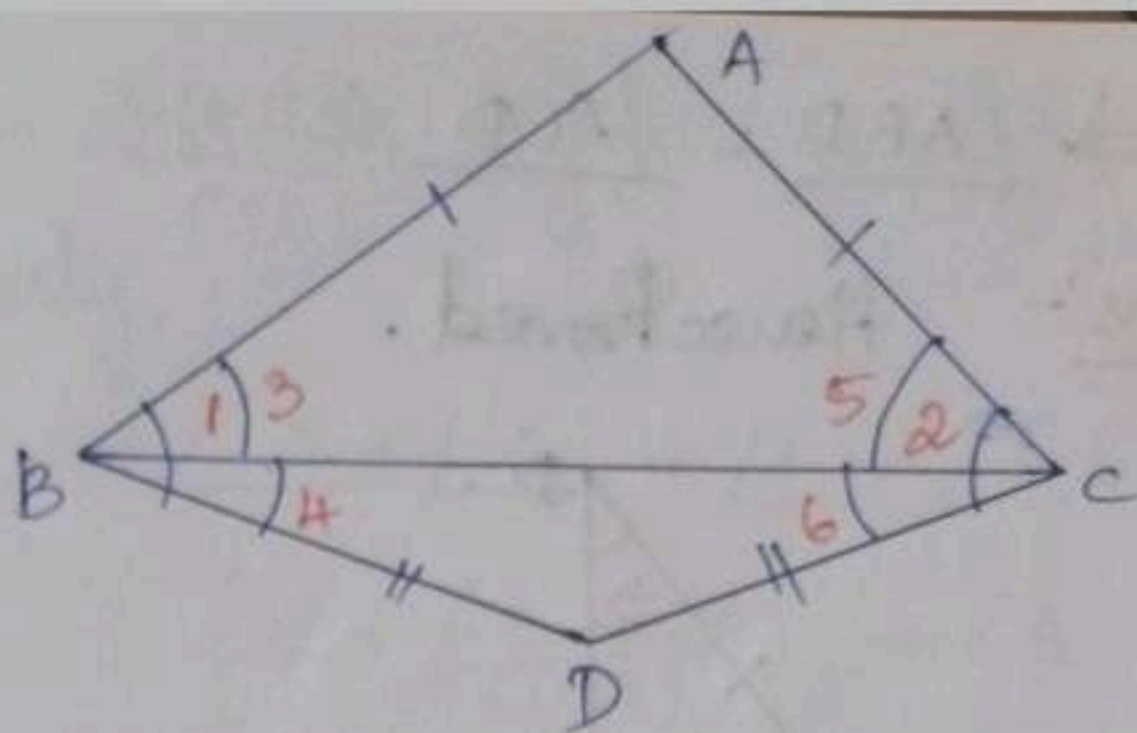
$\Rightarrow \therefore \triangle ABE \cong \triangle ACF$  (By AAS Rule)

$\Rightarrow AB = AC$  (By CPCT)

Ans :- Hence Proved.

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Given:-

$\Delta ABC$  and  $\Delta DBC$  are isosceles triangles on the same BC.

$$\Rightarrow AB = AC \text{ --- (1)}$$

$$\Rightarrow DB = DC \text{ --- (2)}$$

To prove:-

$$\angle ABD = \angle ACD.$$

Solution:-

In  $\Delta ABD$ ,

$$\angle 3 = \angle 5 \text{ [from (1) } \Rightarrow AB = AC \text{ and } \angle (3) \text{ Angles opposite to equal sides are equal].}$$

In  $\Delta ACD$ ,

$$\angle 4 = \angle 6 \text{ [from (2) } \Rightarrow DB = DC \text{ and } \angle (4) \text{ Angles opposite to equal sides are equal].}$$

Also, from the figure,

$$\angle 1 = \angle 3 + \angle 4 \text{ --- (5)}$$

$$\text{and } \angle 2 = \angle 5 + \angle 6 \text{ --- (6)}$$

$$(5) \Rightarrow \angle 1 = \angle 3 + \angle 4$$

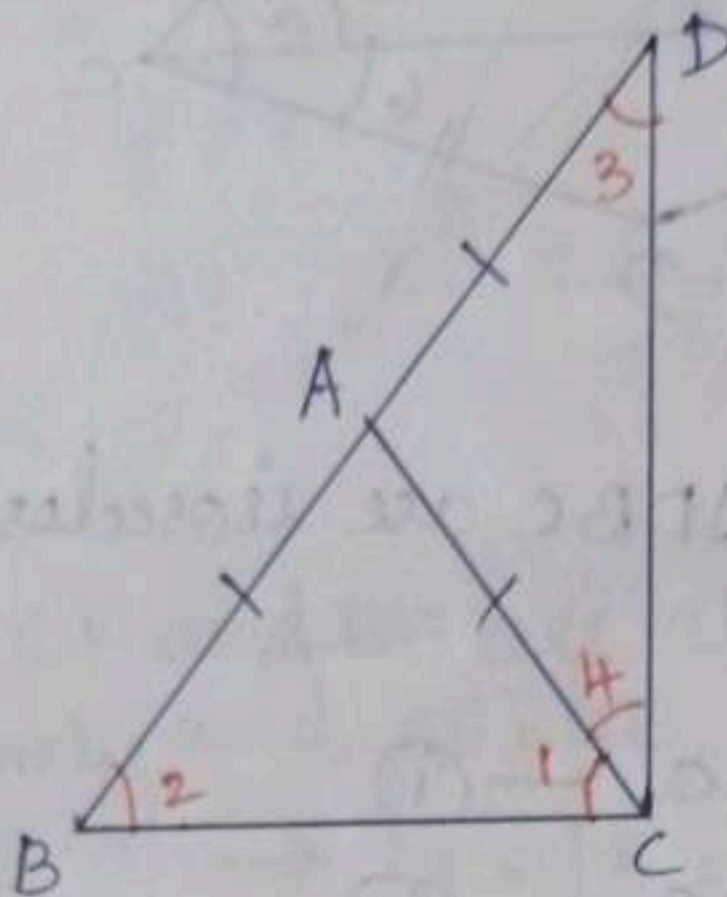
$$\angle 1 = \angle 5 + \angle 6 \text{ (from (3) and (4))}$$

$$\angle 1 = \angle 2 \text{ (from (6))}$$

$\Rightarrow \angle ABD = \angle ACD$

Ans:- Hence Proved.

6)



Given:-  $\Delta ABC$  is isosceles triangle.

$\Rightarrow AB = AC$  — (1)

Also,  $AB = AD$  — (2)

So,  $AC = AD$  — (3) (from (1) and (2))

To prove:-

$\angle BCD = 90^\circ$ .

Solution:-

In  $\Delta ABC$ ,

$\angle 1 = \angle 2$  [from (1) and angles

(4) opposite to equal sides are equal].

In  $\Delta ACD$ ,

$\angle 3 = \angle 4$  [from (3) and angles

(5) opposite to equal sides are equal].

In  $\triangle ABC$ , by using angle sum property, (63)

$$\angle 3 + \angle 2 + \angle 1 + \angle 4 = 180^\circ$$

$$\angle 4 + \angle 1 + \angle 1 + \angle 4 = 180^\circ \text{ (from (4) and (5))}$$

$$2\angle 1 + 2\angle 4 = 180^\circ$$

$$2(\angle 1 + \angle 4) = 180^\circ$$

$$\angle 1 + \angle 4 = \frac{180^\circ}{2}$$

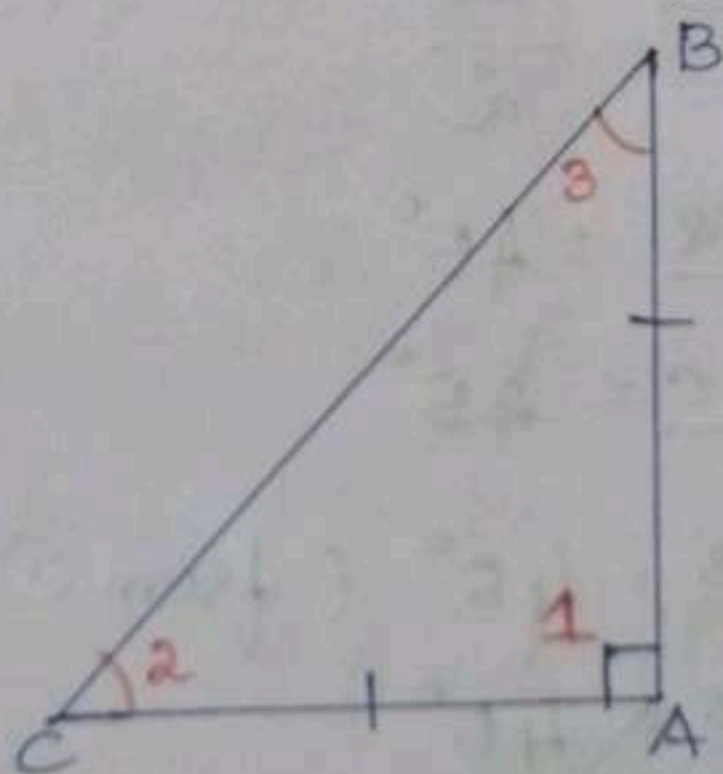
$$\angle 1 + \angle 4 = 90^\circ$$

$$\Rightarrow \angle BCD = 90^\circ$$

$\Rightarrow \angle BCD$  is a right angle

Ans:-

Hence Proved



Let  $\angle A = \angle 1$ ,  
 $\angle B = \angle 3$  and  
 $\angle C = \angle 2$ .

Given:-

$$\angle A = 90^\circ (\angle 1 = 90^\circ)$$

$$AB = AC.$$

To find:-  $\angle B$  and  $\angle C$  ( $\angle 2$  and  $\angle 3$ )

Solution:-

In  $\triangle ABC$ ,  
 $AB = AC$  (Given)

$\Rightarrow \angle 2 = \angle 3$  (Angles opposite to equal sides  
are equal)

In  $\triangle ABC$ , by using Angle sum property,

$$\angle 1 + \angle 2 + \angle 3 = 180^\circ$$

$$\angle 1 + 2\angle 2 = 180^\circ \quad (\because \angle 2 = \angle 3 \text{ from } \textcircled{1})$$

$$90^\circ + 2\angle 2 = 180^\circ \quad (\because \text{Given } \angle 1 = 90^\circ)$$

$$2\angle 2 = 180^\circ - 90^\circ$$

$$2\angle 2 = 90^\circ$$

$$\angle 2 = \frac{90^\circ}{2}$$

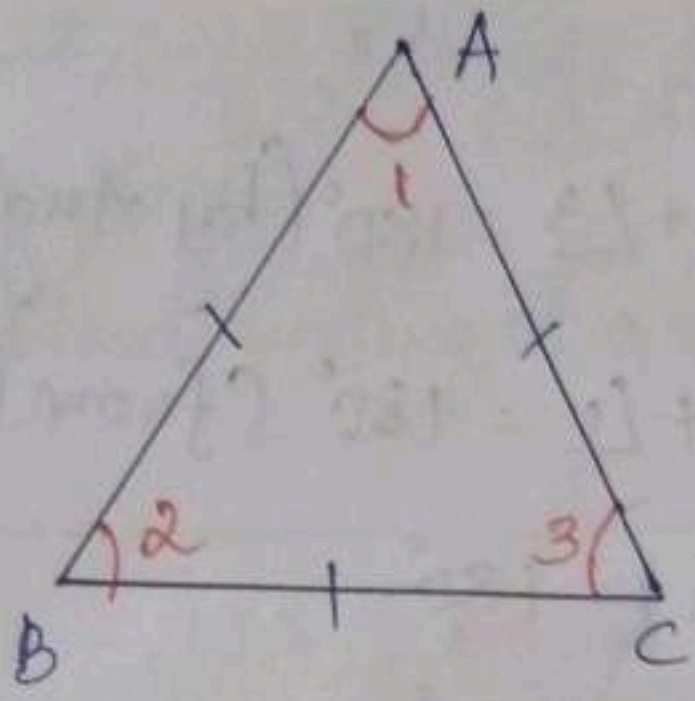
$$\angle 2 = 45^\circ$$

$$\Rightarrow \angle C = 45^\circ$$

$\therefore \angle B = 45^\circ$  (from  $\textcircled{1}$   $\angle 2 = \angle 3$ )

$$\Rightarrow \angle B = 45^\circ$$

Ans:-  $\angle B = \angle C = 45^\circ$



Given :-  
 An equilateral triangle,  $\Delta ABC$   
 $\Rightarrow AB = AC = BC.$

To prove :-  
 Each angle in  $\Delta ABC$  (equilateral triangle) =  $60^\circ$   
 i.e.,  $\angle 1 = \angle 2 = \angle 3 = 60^\circ.$

Solution :-  
 In  $\Delta ABC,$   
 $AB = AC$  (Given)  
 $\angle 3 = \angle 2$  (Angles opposite to equal sides  $\angle 1$  are equal).  
 Also,  $AC = BC$  (Given)  
 $\angle 2 = \angle 1$  (Angles opposite to equal sides  $\angle 3$  are equal).  
 And,  $AB = BC$  (Given)  
 $\angle 3 = \angle 1$  (Angles opposite to equal sides  $\angle 2$  are equal).  
 from ①, ② and ③,  
 $\angle 1 = \angle 2 = \angle 3$  ——— ④



In  $\Delta ABC$ ,

$$\angle 1 + \angle 2 + \angle 3 = 180^\circ \text{ (By Angle Sum Property)}$$

$$\angle 1 + \angle 1 + \angle 1 = 180^\circ \text{ (from (4))}$$

$$3\angle 1 = 180^\circ$$

$$\angle 1 = \frac{180^\circ}{3}$$

$$\angle 1 = 60^\circ$$

$$\Rightarrow \angle 1 = \angle 2 = \angle 3 = 60^\circ \text{ (from (4))}$$

$\Rightarrow$  Each angle in an equilateral triangle =  $60^\circ$

Ans:-

Hence Proved.

## ⇒ Median:-

\* A line segment joining the vertex of the triangle to the midpoint of the opposite side of the triangle is called the median of the triangle.

\* The medians of an equilateral triangle are equal.

\* In a triangle, sum of any 2 sides is greater than the third side.

\* In a triangle, side opposite to greater angle is longer.

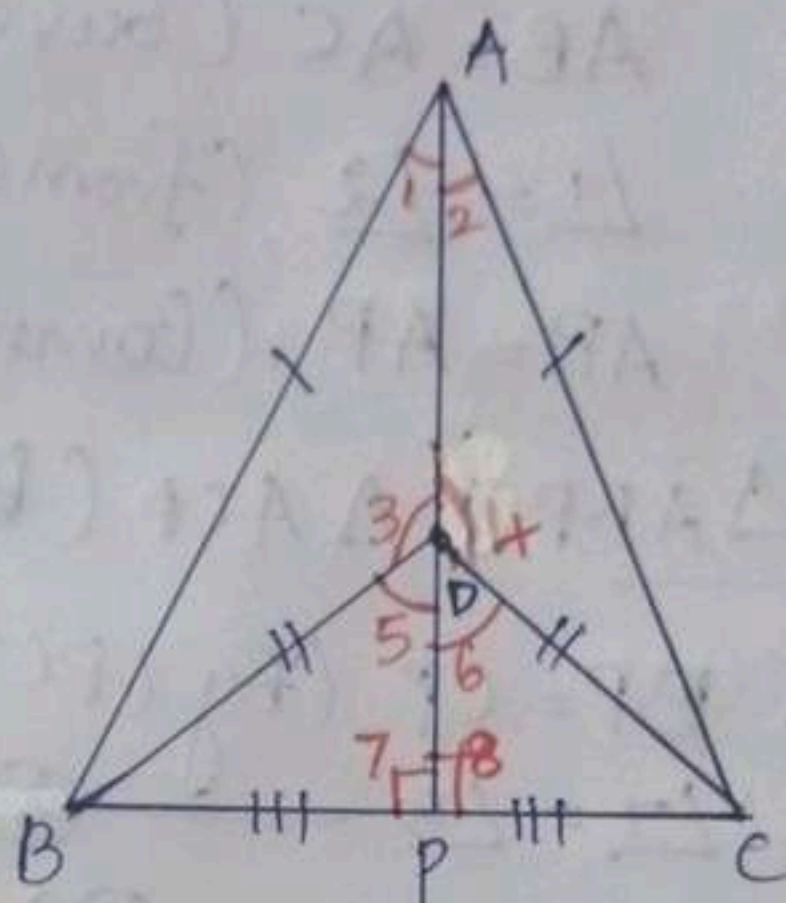
\* In a right angle triangle, the hypotenuse is the longest side.

\* The perimeter of a triangle is greater than the sum of its medians. (67)

\* Three medians meeting at a common point is called centroid.

Exercise 7.3 (Pg. 128)

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Given:-

$\triangle ABC$  is an isosceles triangle

$$\Rightarrow AB = AC.$$

$\triangle BDC$  is an isosceles triangle

$$\Rightarrow BD = CD.$$

To prove:-

i)  $\triangle ABD \cong \triangle ACD$

ii)  $\triangle ABP \cong \triangle ACP$

iii) AP bisects  $\angle A$  as well as  $\angle D$ .

iv) AP is the perpendicular bisector of BC.

Solution:-

i) In  $\triangle ABD$  and  $\triangle ACD$ ,

$$AB = AC \text{ (Given)}$$

$$BD = CD \text{ (Given)}$$

$$AD = AD \text{ (Common)}$$

$\Rightarrow \therefore \triangle ABD \cong \triangle ACD$  (By SSS Rule) (68)

So,  $\angle 1 = \angle 2$  — (1) (By CPCT)

$\angle 3 = \angle 4$  — (2) (By CPCT)

ii) In  $\triangle ABP$  and  $\triangle ACP$ ,

$AB = AC$  (Given)

$\angle 1 = \angle 2$  (from (1))

$AP = AP$  (Common)

$\therefore \triangle ABP \cong \triangle ACP$  (By SAS Rule)

$\Rightarrow$  So,  $BP = CP$  (By CPCT).

$\angle 7 = \angle 8$  — (3) (By CPCT).

iii)  $\angle 3 = \angle 4$  (from (2))

$180^\circ - \angle 3 = 180^\circ - \angle 4$  [Using linear pair angles,

$\angle 5 = \angle 6$

$\angle 3 + \angle 5 = 180^\circ$

$\Rightarrow \therefore AP$  bisects  $\angle D$  and  $\angle 4 + \angle 6 = 180^\circ$ ]

$\therefore \angle 1 = \angle 2$  (from (1))

$\Rightarrow AP$  bisects  $\angle A$  also.

iv)  $\angle 7 = \angle 8$  (from (3))

$\angle 7 + \angle 8 = 180^\circ$

$\angle 7 + \angle 7 = 180^\circ$  ( $\because \angle 7 = \angle 8$ )

$2\angle 7 = 180^\circ$

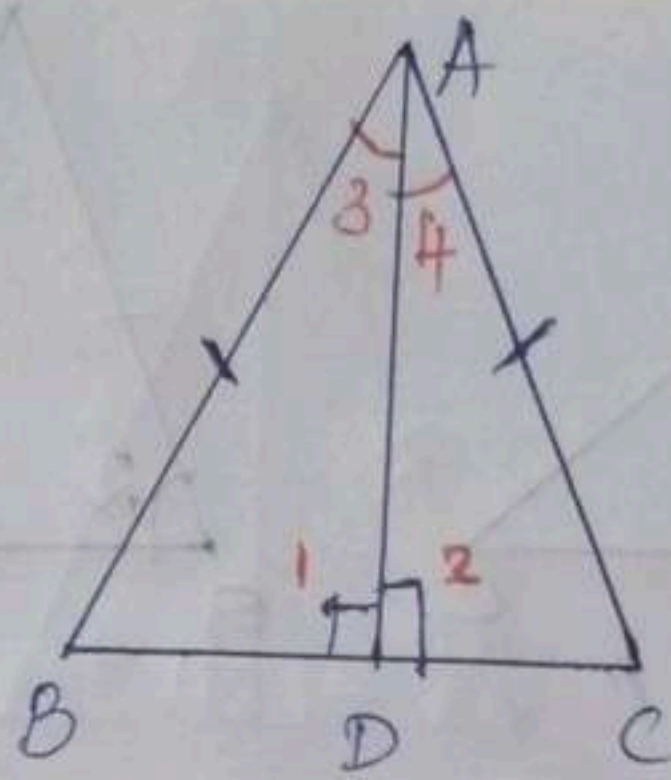
$\angle 7 = \frac{180^\circ}{2} = 90^\circ$

$\angle 7 = \angle 8 = 90^\circ$

Ans

$\Rightarrow$

$AP$  is the perpendicular bisector of  $BC$



Given:-

AD - altitude

$\Rightarrow \angle 1 = \angle 2 = 90^\circ$  ————— (1)

$\Delta ABC$  - isosceles triangle

$\Rightarrow AB = AC$  ————— (2)

To prove:-

i) AD bisects BC

ii) AD bisects  $\angle A$ .

Solution:-

In  $\Delta ABD$  and  $\Delta ACD$ ,

$AB = AC$  (from (2))

$AD = AD$  (Common)

$\angle 1 = \angle 2$  (from (1))

$\therefore \Delta ABD \cong \Delta ACD$  (By RHS rule)

$BD = CD$  (By CPCT)

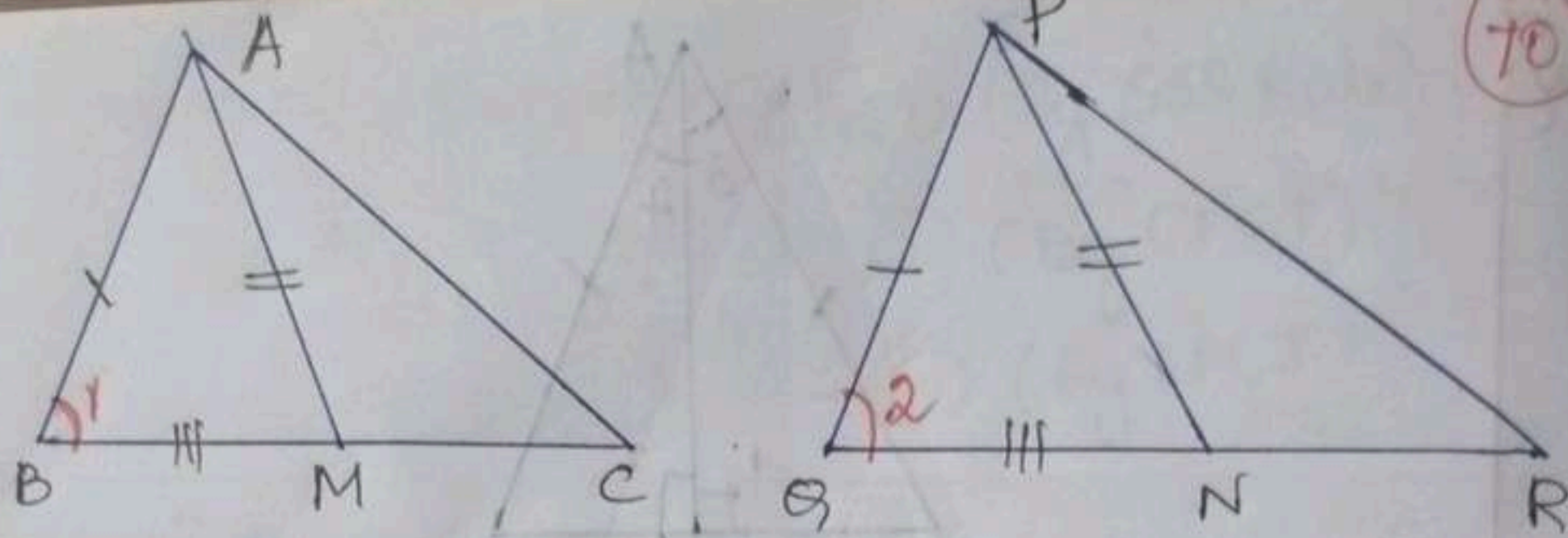
$\Rightarrow \therefore AD$  bisects BC  
Hence Proved.

Also,  $\angle 3 = \angle 4$  (By CPCT)

$\Rightarrow \therefore AD$  bisects  $\angle A$   
Hence Proved.

Ans:-

3)

Given:-

$$AB = PQ$$

$$BC = QR$$

Median  $AM =$  Median  $PN$ .To prove:-

$$i) \Delta ABM \cong \Delta PQN$$

$$ii) \Delta ABC \cong \Delta PQR$$

Solution:-i) In  $\Delta ABM$  and  $\Delta PQN$ ,

$$AB = PQ \text{ (Given)}$$

$$BC = QR \text{ (Given)}$$

$$\therefore \frac{1}{2} BC = \frac{1}{2} QR$$

$$\Rightarrow BM = QN$$

$$AM = PN \text{ (Given)}$$

$$\therefore \Delta ABM \cong \Delta PQN \text{ (By SSS Rule)}$$

$$\Rightarrow \angle 1 = \angle 2 \text{ (By CPCT) — (1)}$$

ii) In  $\Delta ABC$  and  $\Delta PQR$ ,

$$AB = PQ \text{ (Given)}$$

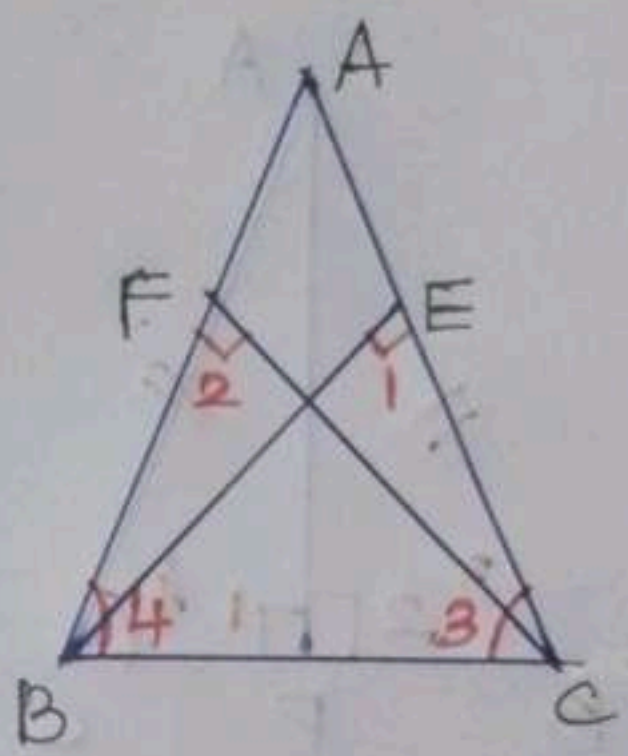
$$\angle 1 = \angle 2 \text{ (from (1))}$$

$$BC = QR \text{ (Given)}$$

$$\Rightarrow \therefore \Delta ABC \cong \Delta PQR \text{ (By SAS Rule)}$$

Ans:-

Hence Proved



Given :-

$BE = CF$  ;  $BE$  and  $CF$  are altitudes .

$\Rightarrow \angle 1 = \angle 2 = 90^\circ$ .

To prove :-

$\triangle ABC$  is isosceles .

Solution :-

In  $\triangle BFC$  and  $\triangle CEB$ ,

$BC = CB$  (Common) (Hypotenuse)

$\angle 1 = \angle 2 = 90^\circ$  (Given)

$CF = BE$  . (Given)

$\therefore \triangle BFC \cong \triangle CEB$  (By RHS Rule)

$\Rightarrow \angle 3 = \angle 4$  (By CPCT)

This gives,

$AB = AC$  (Sides opposite to equal angles are equal).

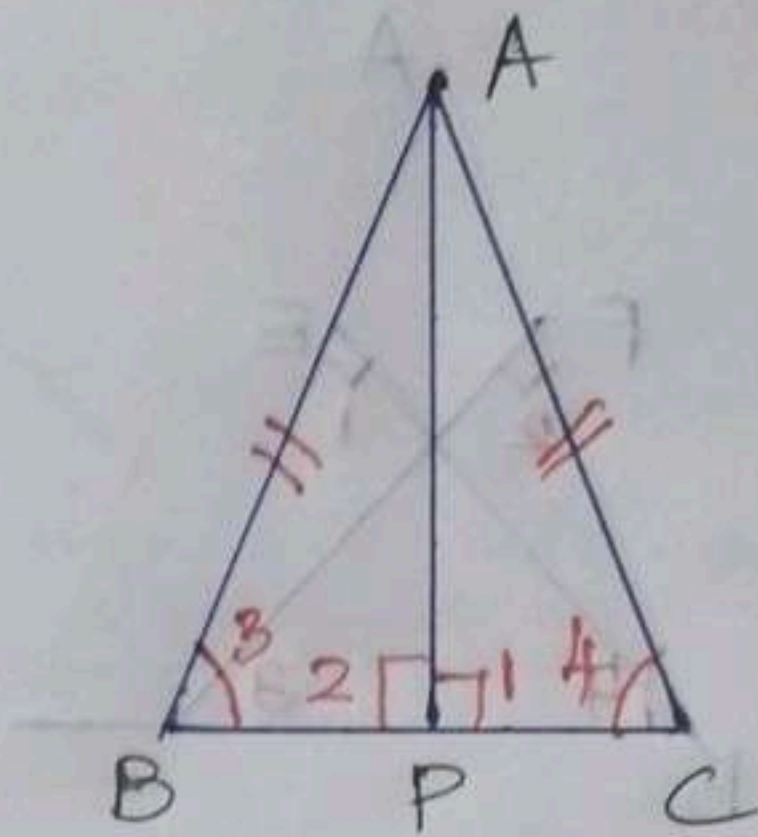
$\Rightarrow \therefore \triangle ABC$  is isosceles triangle

Ans :-

Hence Proved

5)

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Given:- $\triangle ABC$  is isosceles triangle

$$\Rightarrow AB = AC$$

$$AP \perp BC$$

$$\Rightarrow \angle 1 = \angle 2 = 90^\circ$$

To prove:-

$$\angle B = \angle C \quad (\angle 3 = \angle 4)$$

Solution:-In  $\triangle ABP$  and  $\triangle ACP$ ,

$$AB = AC \text{ (Given)}$$

$$\angle 1 = \angle 2 = 90^\circ \text{ (Given)}$$

$$AP = AP \text{ (Common)}$$

$$\therefore \triangle ABP \cong \triangle ACP \text{ (By RHS rule)}$$

$$\Rightarrow \angle 3 = \angle 4 \text{ (By CPCT)}$$

$$\Rightarrow \angle B = \angle C$$

Ans: Hence Proved